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UNIT-5

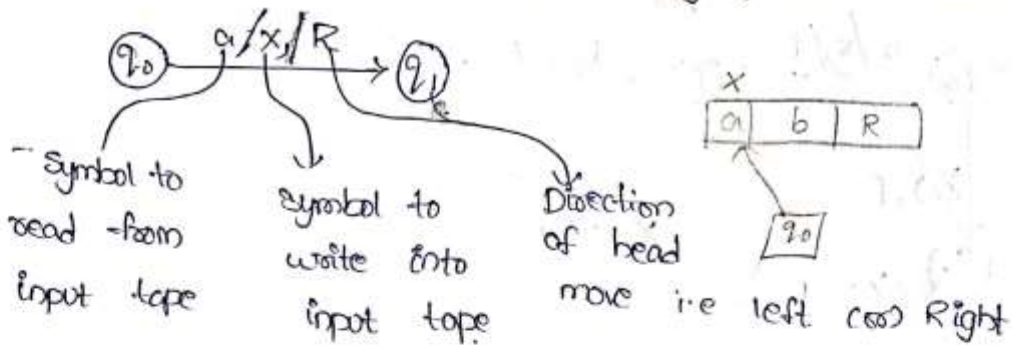
TURING MACHINE

Turing machine contains 7 tuples $(Q, \Sigma, \Gamma, \delta, q_0, F, B)$

- where Q = set of states
- Σ = set of input symbols
- Γ = set of input tape symbols
- δ = Transition function
- q_0 = initial state
- F = final state
- B = Blanks symbol.

where δ can be defined as $\delta: Q \times \Gamma \rightarrow Q \times \Gamma \times (L/R)$

The Transition Diagram:- The transition function can be represented in the form of graphical notation.

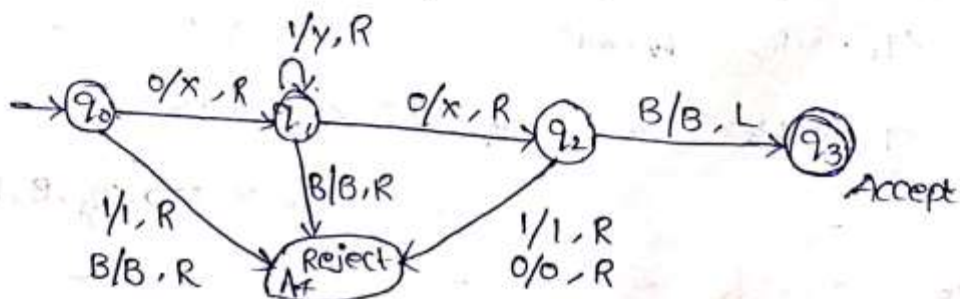
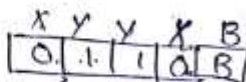
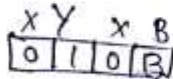
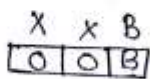
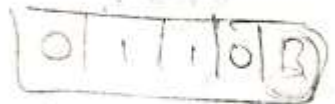


1) Design a Turing machine for $L = 01^*0$

$L = 01^*0$

01^*0

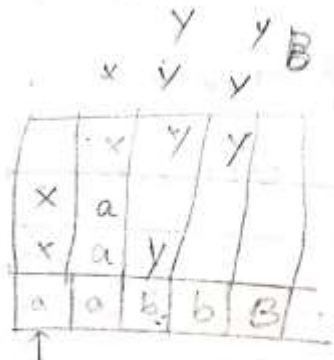
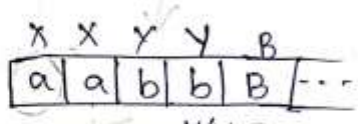
$L = \{00, 010, 0110, 01110, \dots\}$



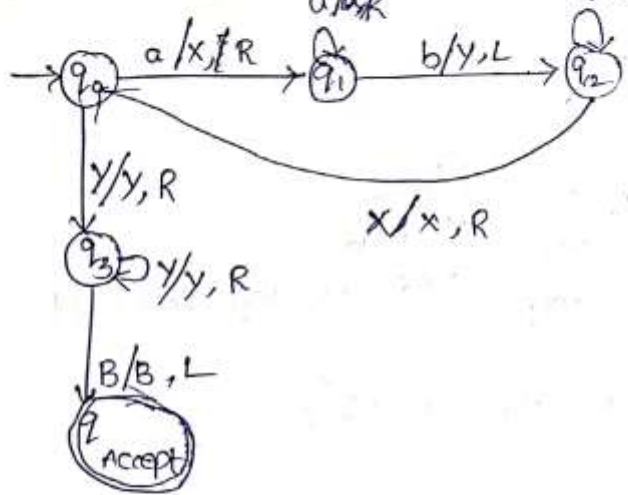
Start State	0	1	x	y	B
q ₀	<q ₁ , x, R>	<q ₄ , 1, R>	-	-	<q ₄ , B, R>
q ₁	<q ₂ , x, R>	<q ₁ , y, R>	-	-	<q ₄ , B, R>
q ₂	<q ₄ , 0, R>	<q ₄ , 1, R>	-	-	<q ₃ , B, L>
q ₃	-	-	-	-	-
q ₄	-	-	-	-	-

*
**
Design a Turing Machine for language $L = \{a^n b^n / n \geq 1\}$

$L = \{ab, aabb, aaa bbb, \dots\}$



each 'a' is converted to 'b'

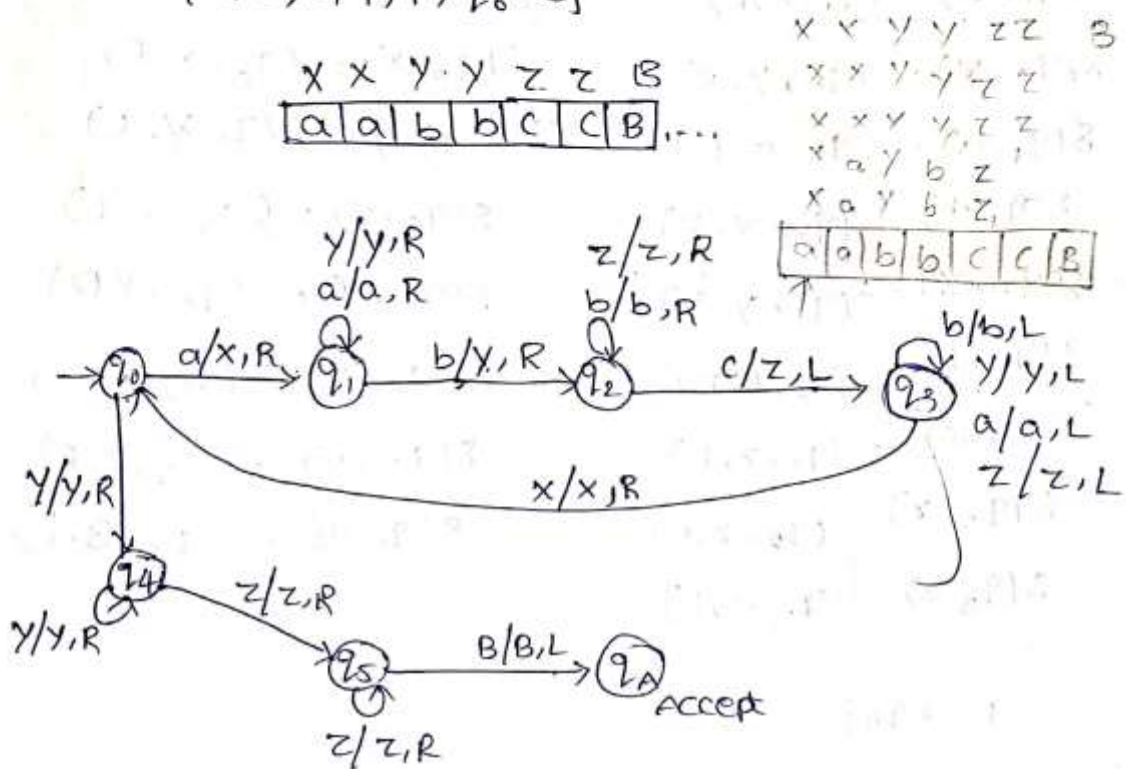


Tape symbol State	a	b	x	y	B
→ q ₀	<q ₁ , x, R>	-	-	<q ₃ , y, R>	-
q ₁	<q ₁ , a, R>	<q ₂ , y, L>	-	<q ₁ , y, R>	-
q ₂	<q ₂ , a, L>	-	<q ₀ , x, R>	<q ₂ , y, L>	-
q ₃	-	-	-	<q ₃ , y, R>	<q ₄ , B, L>
* q ₄ Accept	-	-	-	-	-

8
 Design Turing machine for $L = \{a^n b^n c^n / n \geq 1\}$

$L = \{abc, aabbcc, aaabbbccc, \dots\}$

$TM = \{Q, \Sigma, \Gamma, F, q_0, B\}$



Tape symbol / state	a	b	c	x	y	z	B
$\rightarrow q_0$	$\langle q_1, x, R \rangle$	-	-	-	$\langle q_4, y, R \rangle$	-	-
q_1	$\langle q_1, a, R \rangle$	$\langle q_2, y, R \rangle$	-	-	$\langle q_1, y, R \rangle$	-	-
q_2	-	$\langle q_2, b, R \rangle$	$\langle q_3, z, L \rangle$	-	-	$\langle q_2, z, R \rangle$	-
q_3	$\langle q_3, a, L \rangle$	$\langle q_3, b, L \rangle$	-	$\langle q_0, x, R \rangle$	$\langle q_3, y, L \rangle$	$\langle q_3, z, L \rangle$	-
q_4	-	-	-	-	$\langle q_4, y, R \rangle$	$\langle q_5, z, R \rangle$	-
q_5	-	-	-	-	-	$\langle q_5, z, R \rangle$	$\langle q_A, B, L \rangle$
q_A	-	-	-	-	-	-	-

TM: $M = \{Q, \Sigma, \Gamma, \delta, q_0, B, F\}$

$Q = \{q_0, q_1, q_2, q_3, q_4, q_5, q_A\}$

$\Sigma = \{a, b\}$

$\Gamma = \{a, b, c, x, y, z, B\}$

$\delta(q_0, a) = (q_1, x, R)$

$\delta(q_3, b) = (q_3, b, L)$

$\delta(q_0, y) = (q_4, y, R)$

$\delta(q_3, x) = (q_0, x, R)$

$\delta(q_1, a) = (q_1, a, R)$

$\delta(q_3, y) = (q_3, y, L)$

$\delta(q_1, b) = (q_2, y, R)$

$\delta(q_3, z) = (q_3, z, L)$

$\delta(q_1, y) = (q_1, y, R)$

$\delta(q_4, y) = (q_4, y, R)$

$\delta(q_2, b) = (q_2, b, R)$

$\delta(q_4, z) = (q_5, z, R)$

$\delta(q_2, c) = (q_3, z, L)$

$\delta(q_5, z) = (q_5, z, R)$

$\delta(q_2, z) = (q_2, z, R)$

$\delta(q_5, B) = (q_A, B, L)$

$\delta(q_3, a) = (q_3, a, L)$

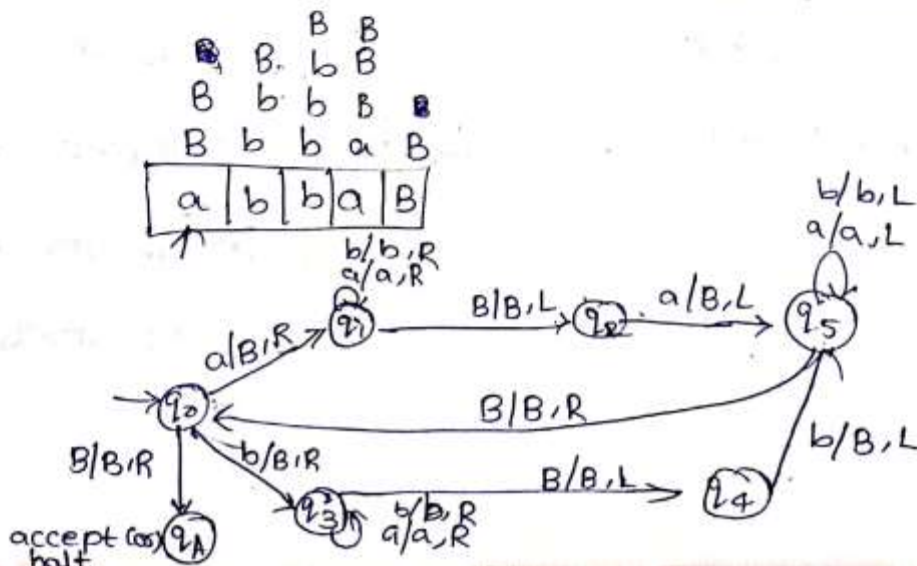
$F = \{q_A\}$

Design Turing Machine for $L = \{ww^R \mid w \in (a, b)^*\}$

Give $L = \{ww^R \mid w \in (a, b)^*\}$

It is a even length palindrome

$L = \{aa, bb, abba, baab, abbbba, abaabba, \dots\}$



State \ Tape Symbols	a	b	B
$\rightarrow q_0$	$\langle q_1, B, R \rangle$	$\langle q_3, B, R \rangle$	$\langle q_A, B, R \rangle$
q_1	$\langle q_1, a, R \rangle$	$\langle q_1, b, R \rangle$	$\langle q_2, B, L \rangle$
q_2	$\langle q_5, B, L \rangle$	-	-
q_3	$\langle q_3, a, R \rangle$	$\langle q_3, b, R \rangle$	$\langle q_4, B, L \rangle$
q_4	-	$\langle q_5, B, L \rangle$	-
q_5	$\langle q_5, a, L \rangle$	$\langle q_5, b, L \rangle$	$\langle q_0, B, R \rangle$
* q_A	-	-	-

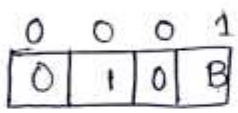
ii) abba

q_0 a b b a B
 T B q_1 b b a B
 T B b q_1 b a B
 T B b b q_1 a B
 T B b b a q_1 B
 T B b b q_2 a B
 T B b q_5 b B B
 T B q_5 b b B B
 T q_5 B b b B B
 T B q_0 b b B B
 T B B q_3 b B B
 T B B b q_3 B B
 T B B q_4 b B B
 T B q_5 B B B B
 T B B q_0 B B B
 T B B B q_A B B
 accept

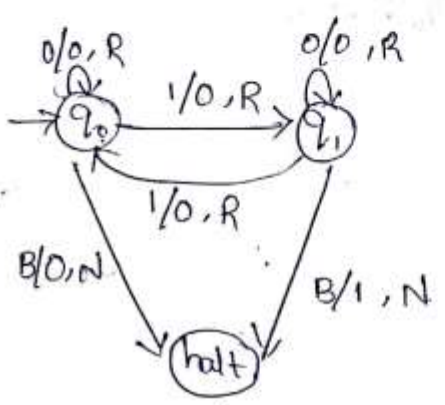
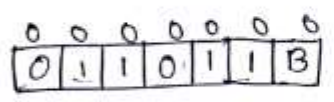
5) Design Turing Machine for 'parity checker' that outputs '0' depending on w

1's odd - 1

1's even - 0



Transition Table



odd 1's 010

even 1's 1010

⊢ q₀ 0 1 0 B

⊢ q₀ 1 0 1 0 B

⊢ 0 q₀ 1 0 B

⊢ 0 q₁ 0 1 0 B

⊢ 0 0 q₀ 0 B

⊢ 0 0 q₁ 1 0 B

⊢ 0 0 0 q₁ B

⊢ 0 0 0 q₀ 0 B

⊢ 0 0 0 1 halt

⊢ 0 0 0 0 q₀ B

⊢ 0 0 0 0 0 halt

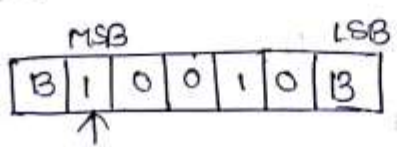
Design TM for 2's complement

I/P 10010

1's 01101

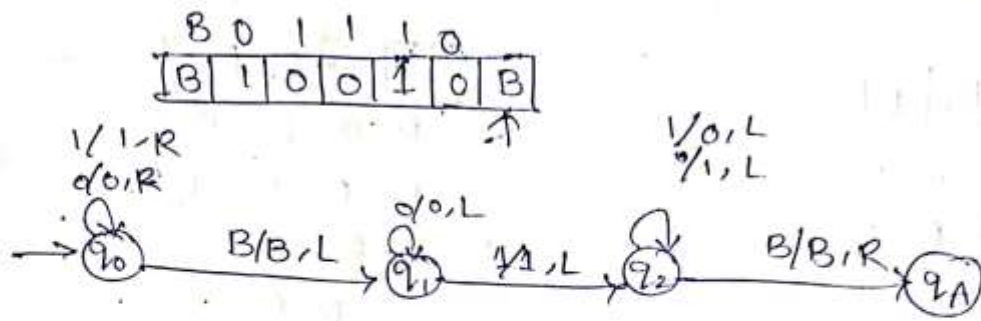
+1

2's 01110



accept
halt

First of all we have to move LSB then upto



$q_0: B10010B$

$q_1: B010010B$

$q_2: B100010B$

$q_0: B100010B$

$q_1: B100010B$

$q_0: B100010B$

$q_1: B1001000B$

$q_1: B1001000B$

$q_1: B1000100B$

$q_2: B1000100B$

$q_2: B1000100B$

$q_2: B0111100B$

$q_2: B0111100B$

$q_2: B0111100B$

$q_2: B0111100B$

* (Addition of two integers. Design TM)
 design Turing Machine for to accept set of all the
 palindromes.

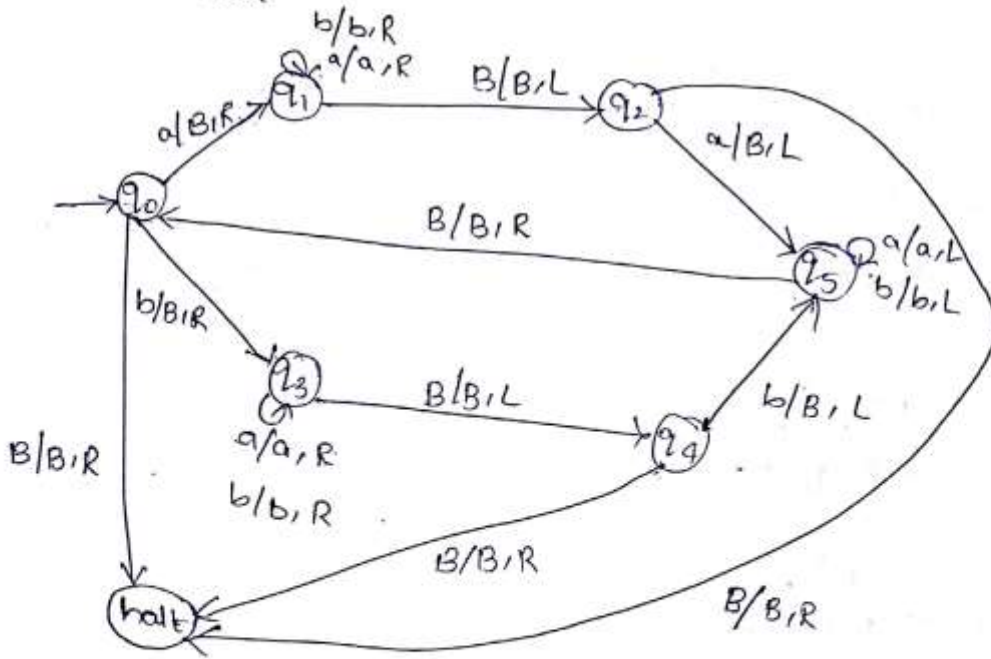
Sol:

a	b	a	B
---	---	---	---

B b a B
 B b B B
 B B B
 halt

b	a	b	B
---	---	---	---

B a b B
 B a B B
 B B B
 B B
 halt

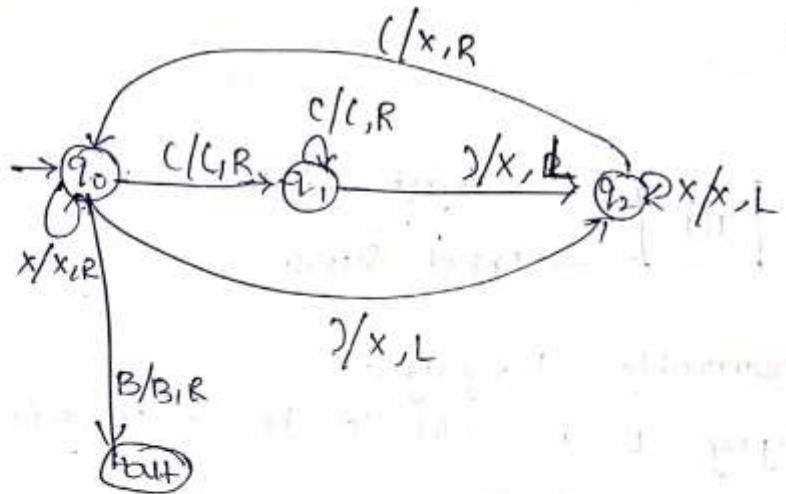
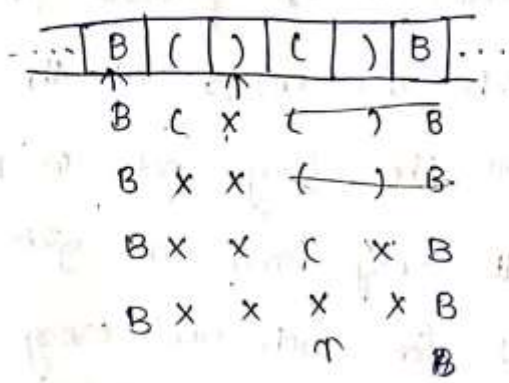


bab

- ⊢ q₀ babB
- ⊢ B q₃ abB
- ⊢ B a q₃ bB
- ⊢ B a b q₃ B
- ⊢ B a B q₄ bB
- ⊢ B q₅ a B B
- ⊢ q₅ B a B B
- ⊢ B q₀ a b B
- ⊢ B B q₁ B B
- ⊢ B q₂ B B B
- ⊢ B B q₁ B B

Design Turing Machine for parenthesis checking

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- $q_0 (()) B$
- $(q_1 ()) B$
- $((q_1)) B$
- $((q_2 x) B$
- $(q_2 (x) B$
- $(x q_0 x) B$
- $(x x q_0) B$
- $(x q_0 x x B$
- $(q_2 x x x B$
- $q_2 (x x x B$
- $x q_0 x x x B$
- $x x q_0 x x B$
- $x x x q_0 x B$
- $x x x x q_0 B$

$x x x x B \vdash$

Types of Grammars - Chomsky Hierarchy:

Linguist Noam Chomsky defined a hierarchy of languages in terms of complexity. This four level hierarchy, called the Chomsky hierarchy, corresponds to four classes of machines.

The Chomsky hierarchy classifies grammars according to form of their productions into the following four levels.

- (1) Type 0 grammars - unrestricted grammar
- (2) Type 1 grammars - context sensitive grammar
- (3) Type 2 grammars - context free grammar
- (4) Type 3 grammars - regular grammars.

(1) Type-0 grammars - Unrestricted Grammars (URG)

These grammars include all formal grammars. In URGs, all the productions are of the form $\alpha \rightarrow \beta$, where α and β may have any number of terminals and non-terminals, i.e., no restrictions on either side of productions.

Every grammar is included in it if it has at least one non-terminal on the left hand side.

Ex:-
 $aA \rightarrow abCB$
 $aA \rightarrow bAA$
 $bA \rightarrow a$
 $S \rightarrow aAb|c$

or ~~(A, B, C)~~

They generate exactly all languages that can be recognized by a Turing machine. The language that is recognized by a Turing machine is defined as set of all the strings on which it halts. These languages

are also known as the recursively enumerable languages. (2)

(2) Type 1 Grammar - Context Sensitive Grammars: (CSG)

These grammars define the context-sensitive languages. In Context Sensitive grammar, all the productions are of the form $\alpha \rightarrow \beta$, where length of α is less than or equal to the length β . i.e. $|\alpha| \leq |\beta|$. α and β may have any number of terminals and non-terminals. These languages are exactly all the languages that can be recognized by linear bound automata.

Ex: $|\alpha| \leq |\beta| \quad \alpha \rightarrow \beta$
 $aAbcD \rightarrow abcDbcD$

(3) Type 2 Grammar - Context-free Grammar (CFG)

These grammars define the context-free languages. These are defined by rules of the form $\alpha \rightarrow \beta$ with $|\alpha| \leq |\beta|$ where $|\alpha| = 1$ and α is non-terminal and β is a string of terminals and non-terminals. We can replace α by β regardless of where it appears. Hence the name context free grammar.

These languages are exactly those languages that can be recognized by a pushdown automaton. CFG defines the syntax of all programming languages.

Ex: (i) $S \rightarrow aS | Sa | a$
(ii) $S \rightarrow aAA | bBB | \epsilon$

(4) Type 3 grammars - regular grammars:

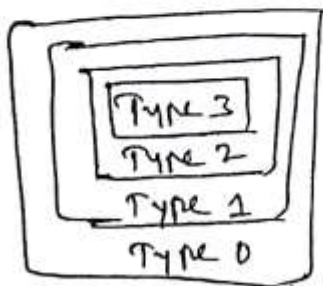
These grammars generate the regular languages. Such a grammar restricts its rules to a single non-terminal on the LHS. The RHS consists of either a single terminal or string of terminals with single non-terminal on left or right end.

Ex: $A \rightarrow aA|a$ — right linear grammar $\alpha = \{V\}$
 $A \rightarrow Aa|a$ — left linear grammar $\beta = \{V\}^* \cup \{T\}^*$

Every regular language is context free, every context-free language is context-sensitive and every context-sensitive language is recursively enumerable.

Table: Chomsky's hierarchy

<u>Grammar</u>	<u>Language</u>	<u>Automaton</u>	<u>production rules</u>
Type 0	Recursively enumerable	Turing machine	$\alpha \rightarrow \beta$ no restrictions on α, β α should have at least one non-terminal
Type 1	Context-sensitive	Linear bounded Automate	$\alpha \rightarrow \beta$ $ \alpha \leq \beta $
Type 2	Context-free	Push down automata	$\alpha \rightarrow \beta$ $ \alpha = 1$
Type 3	Regular	Finite state automaton	$\alpha \rightarrow \beta$ $\alpha = \{V\}$ $\beta = \{V\}^* \cup \{T\}^*$ $= T^* \{V\}$ $= T^*$



Chomsky hierarchy of grammars

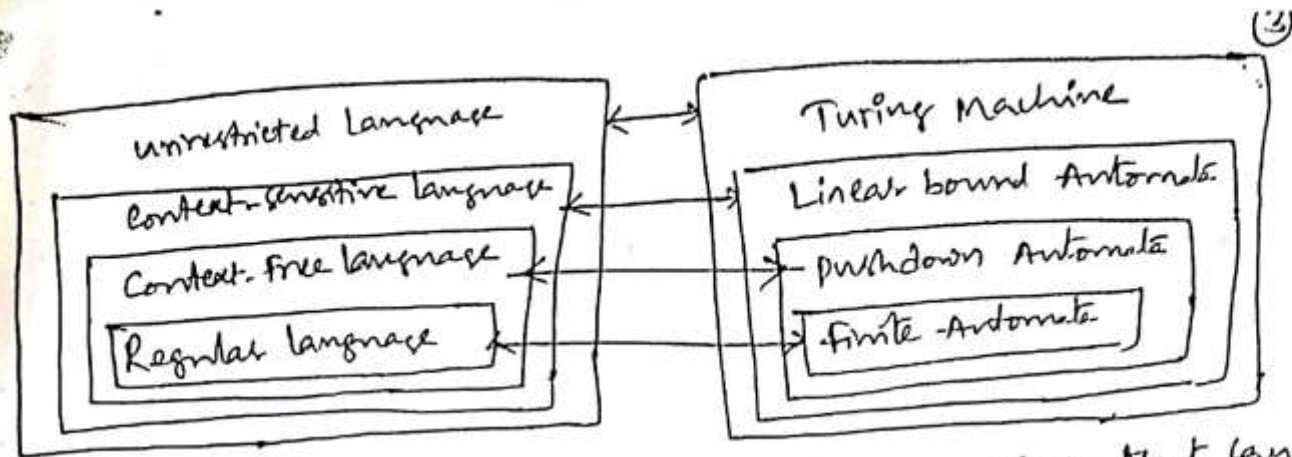


Fig: The hierarchy of languages and the machine that can recognize the same is shown above fig.

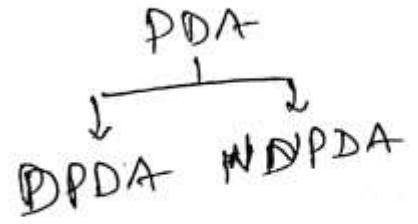
- Every RL is context free, every CFL is context-sensitive and every CSL is unrestricted. So the family of regular language can be recognized by any machine.
- CFLs are recognized by pushdown automata, linear bounded automata and Turing machines.
- CSLs are recognized by Linear bounded automata and Turing machines.
- Unrestricted languages are recognized by only Turing machines.

(4)

Push Down Automata (PDA)

①

PDA - FA + Stack
memory element



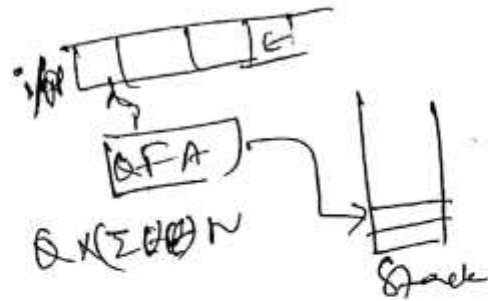
$$PDA = (Q, \Sigma, \delta, q_0, Z_0, F, \Gamma)$$

(T.M.)

- Q = finite set of states
- Σ = input symbols
- δ = Transition function
- q_0 = initial state
- Z_0 = bottom of the stack
- F = set of final states
- Γ = stack alphabet.

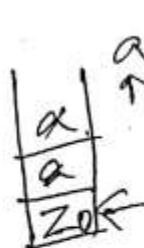
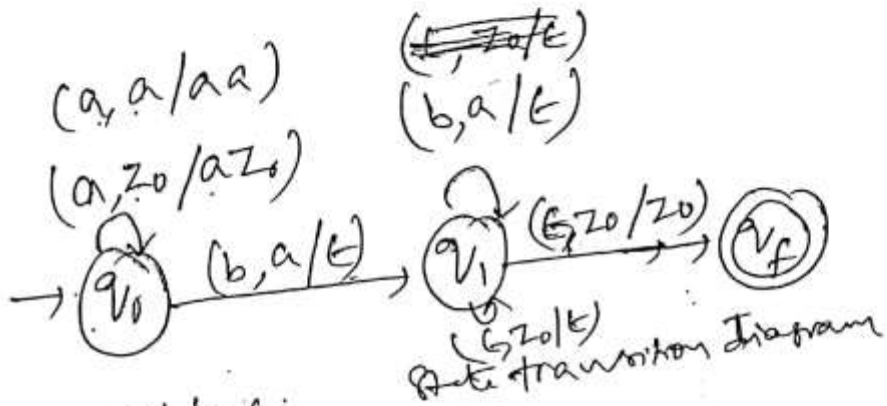
$$DPDA: \delta: \underbrace{Q \times \Sigma}_{\text{state}} \times \underbrace{\{ \epsilon \}}_{\text{input}} \times \underbrace{\Gamma}_{\text{top}} \rightarrow \underbrace{Q \times \Gamma}_{\text{top}}$$

$$NDPDA: \delta: \underbrace{Q \times \Sigma}_{\text{state}} \times \underbrace{\{ \epsilon \}}_{\text{input}} \times \underbrace{\Gamma}_{\text{top}} \rightarrow \underbrace{Q \times \Gamma}_{\text{top}}^*$$



Ex: $a^n b^n | n \geq 1$

a a b b ϵ
 ↑ ↑ ↑ ↑ ↑



a a b b ϵ
 ↑ ↑ ↑ ↑ ↑

$$\begin{aligned} \delta(q_0, a, z_0) &= (q_0, aa) \\ \delta(q_0, a, a) &= (q_0, aa) \\ \delta(q_0, b, a) &= (q_1, \epsilon) \\ \delta(q_1, b, a) &= (q_1, \epsilon) \\ \delta(q_1, \epsilon, z_0) &= (q_f, z_0) \text{ or } (q_f, \epsilon) \end{aligned}$$

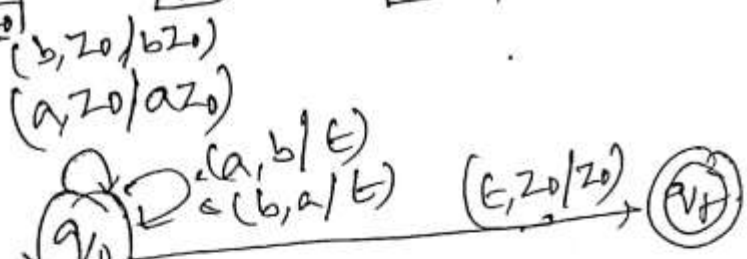
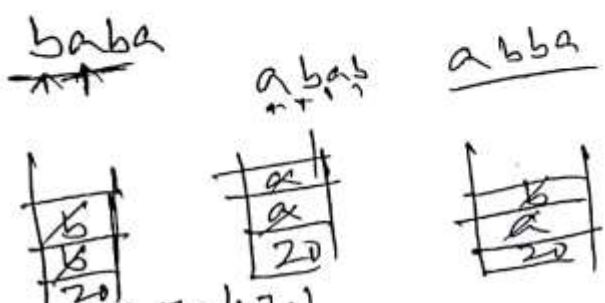
accept by final state Acceptance by empty stack

transitions function

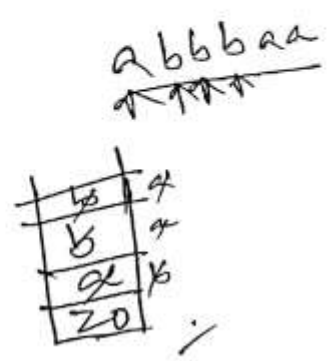
PDA → final state
 → Empty stack

$|w|/n_a(w) = n_b(w)$ [number of a 's must be equal],
DPDA

Ex: $a b$ $b b a a$
 $a a b b$ $b a b a$
 $a b a b$ -

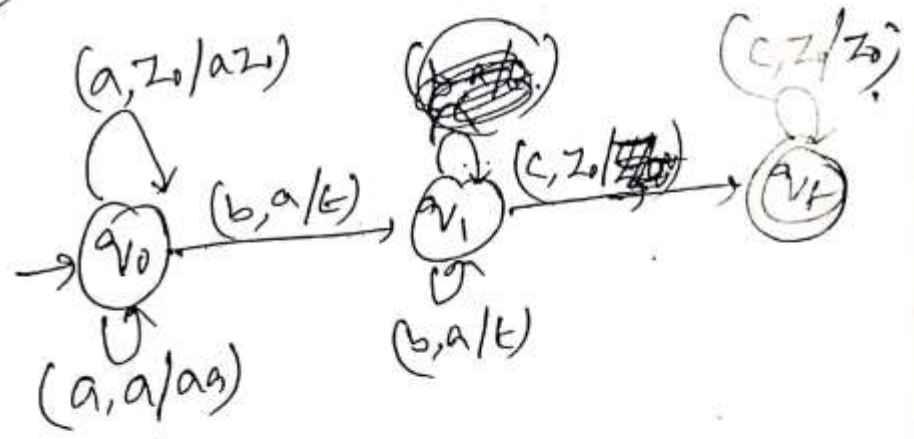


$(a, a / aa)$
 $(b, b / bb)$



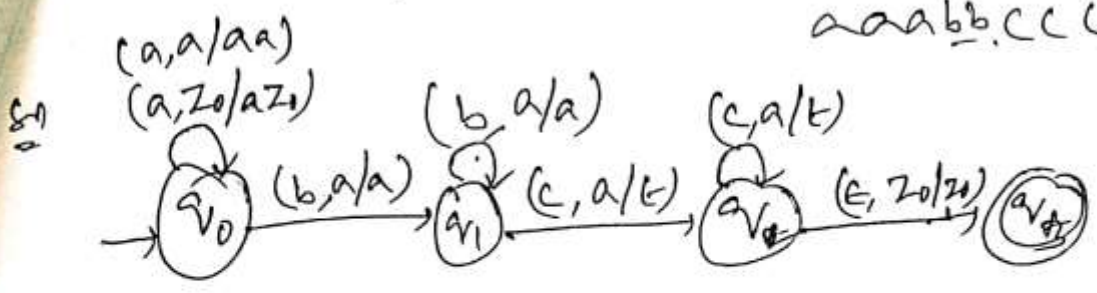
$a^n b^n c^m \quad |n, m \geq 1|$

$a^n \rightarrow$ push
 $b^n \rightarrow$ pop
 $c^m \rightarrow$ in final state

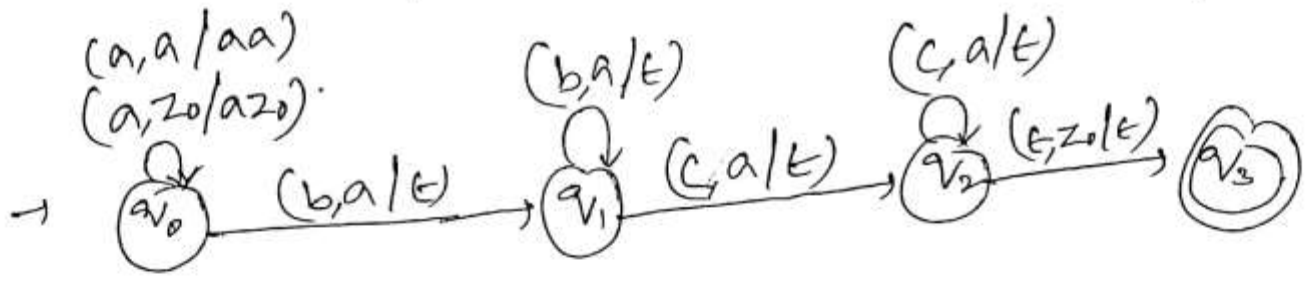


$a^n b^m c^n \mid n, m \geq 1$ dont want cont $\geq b$

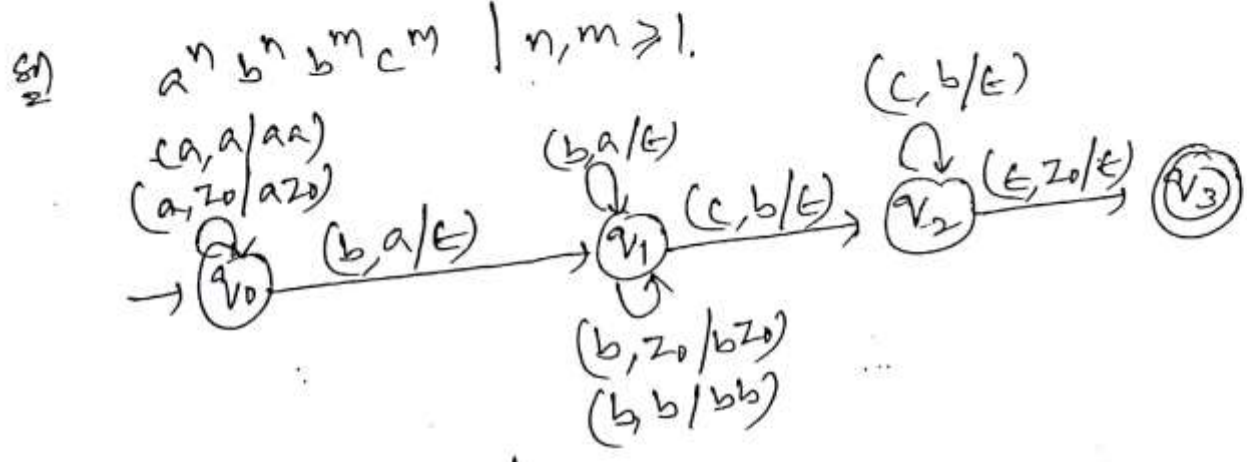
aaab~~b~~.ccc



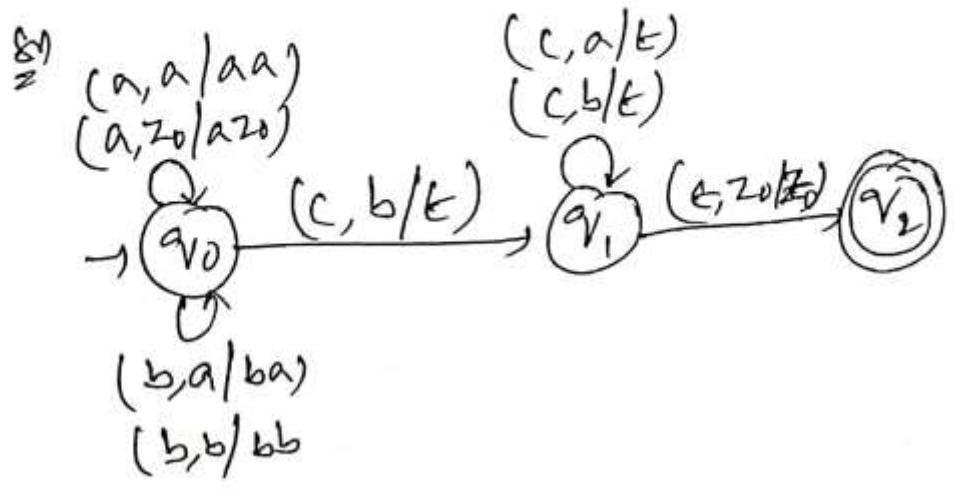
$a^{m+n} b^m c^n \mid m, n \geq 1$



$a^n b^{m+n} c^m \mid n, m \geq 1$



$a^n b^m c^{n+m} \mid n, m \geq 1$



$a^n b^m c^m d^n \mid n, m \geq 1$

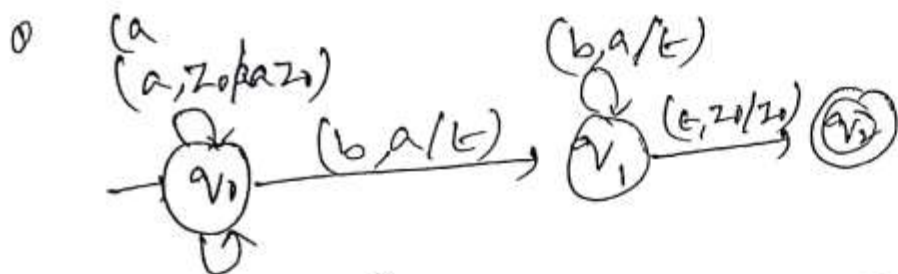
$a^n b^m c^m d^n \mid n, m \geq 1$

$a^n b^m c^n d^m \mid n, m \geq 1$ X is not CF
not PDA

$a^n 2^n \mid n \geq 1$

$a^n b^{2n} \mid n \geq 1$

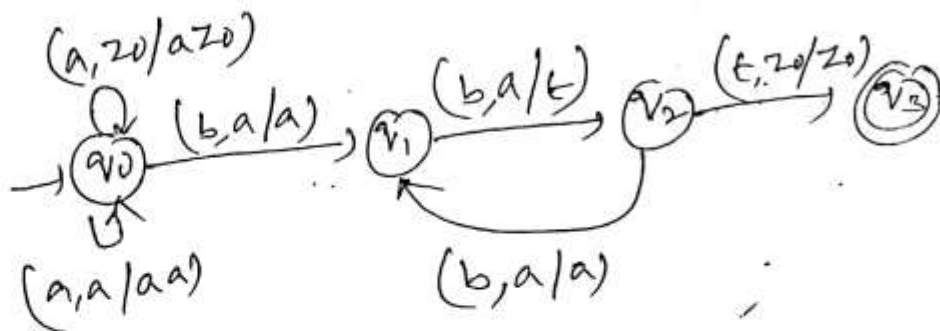
two solutions $\left\{ \begin{array}{l} \text{for every } a - \text{push two } a\text{'s.} \\ \text{for every } b - \text{pop two } b\text{'s} \end{array} \right.$



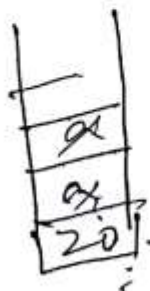
$b b, b b, \dots$

(a, a/aaa)

for every b



$a a b b b b \epsilon$



Ex: $a^n b^n c^n \mid n \geq 1$ ~~X~~ \rightarrow PDA

one possibility

$a a b b c c$
 $\uparrow \uparrow \uparrow \uparrow \uparrow \uparrow$



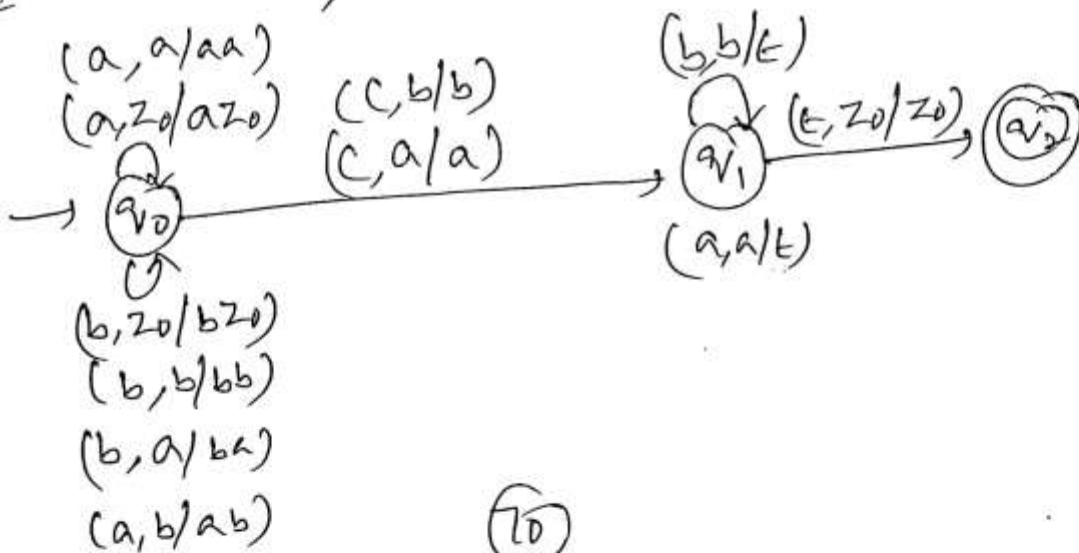
~~$a b^2 b a$~~

for every $a \rightarrow$ two a 's push



Ex: ~~W~~ $W \subseteq (a, b)^+$

Ex: $abcba, abbcbba$



z_0

Ex: $W \subseteq (a+b)^+$ \rightarrow whenever

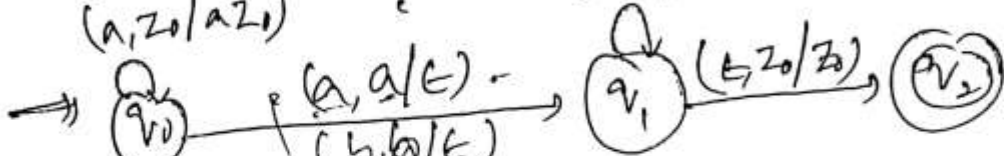
Ex: $\frac{aba}{a} \frac{aba}{b}$



NDPDA $w w^R (w \in (a+b)^+)$

$(b, z_1 / b z_1)$
 $(a, z_0 / a z_0)$

$(b, b / \epsilon)$
 $(a, a / \epsilon)$



$(a, b / ab)$

$(b, a / ba)$

$(a, a / aa)$ } assume.

$(b, b / bb)$

abba

Instantaneous Config

aaag
~~Instantaneous Config~~
 $(q_0, aaaa, z_0)$

(q_0, aaa, az_0)

no letter | centre

(q_0, aa, aaz_0) (q_1, aa, z_0)
 X

no letter | centre

$(q_0, a, aaaaz_0)$ (q_1, a, aaz_0)
 no letter | centre
 $(q_0, \epsilon, aaaaaz_0)$ (q_0, ϵ, aaz_0) (q_1, ϵ, z_0)
 X X (q_2, z_0)
 ✓

~~abab~~

baa aab



a/a/a a
 w/wR

NDPDA

Center has pop come
 Center not come
 push



change \rightarrow getting \rightarrow center

a/b/ba

UNIT-VI

Language decidability

* Introduction

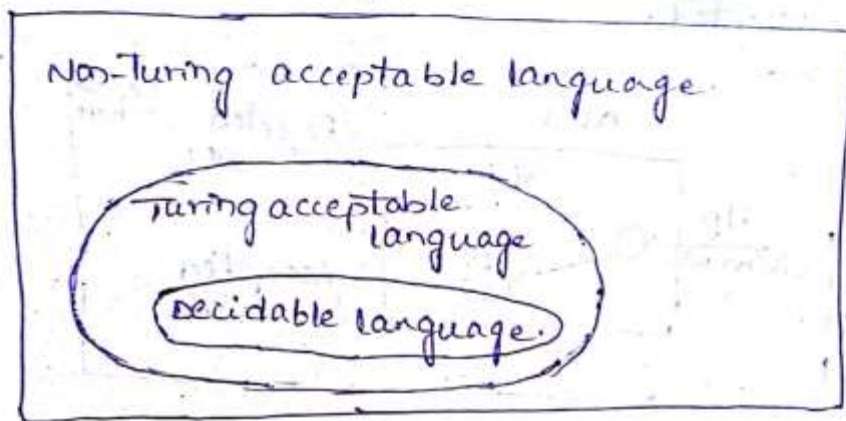
* Examples

Introduction:-

• Decidable problem:-

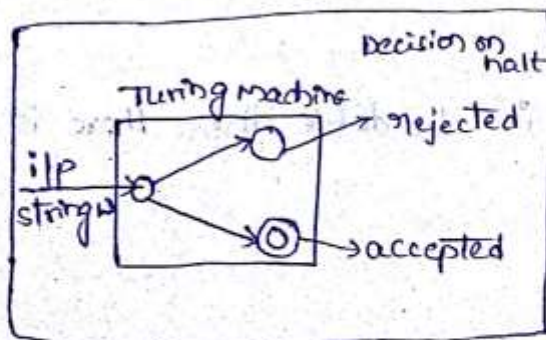
* A language is called Decidable (or) recursive if there is a Turing machine which accepts and halts on every ip string "w".

* Every decidable language is a Turing acceptable.



* A decision problem P is decidable if the language L of all "YES" instances to P is decidable.

* For a decidable language, for each ip string, the Turing machine halts either at the accept (or) the reject state.



Examples:-

1) Find out whether the following problem is decidable (or) not.

Is a number m prime?

sol:- Prime numbers = $\{2, 3, 5, 7, 11, 13, 17, 19, \dots\}$

divide the number m by all the numbers b/w

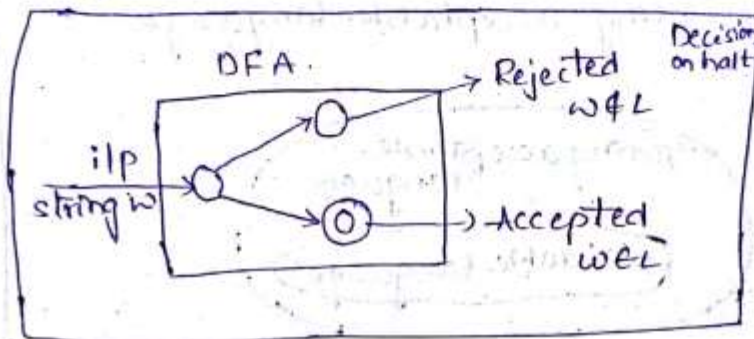
2 and m_2 starting from 2.

If any of these numbers produce a remainder 0, then it goes to the rejected state, otherwise it goes to the accepted state. So, here the answer could be made by YES (or) NO.

Hence, it is a decidable problem.

2) Given a Regular language 'L' and string 'w', how can we check if $w \in L$.

Sol:- Take the DFA that accepts 'L' and check if 'w' is accepted.



Some more decision problems are

i) Does DFA accept the empty language.

ii) Is $L_1 \cap L_2 = \phi$ for regular sets.

iii) If a language L is decidable then its complement L^c is also decidable.

iv) If a language is decidable then there is a Turing machine for it.

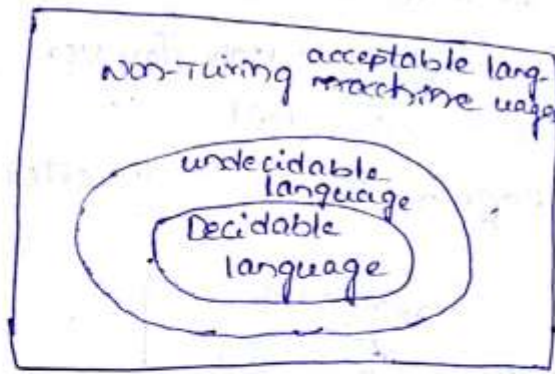
Undecidable problems:-

Introduction:-

* For an undecidable language there is no TM which accepts the language and makes a decision for every ilp string 'w'.

* A decision problem 'p' is called undecidable if the language 'L' of all 'yes' instances to 'p' is not decidable.

undecidable languages are not recursive languages but, sometimes they may be recursive-enumerable languages.



Examples:-

- i) the halting problem of Turing machine.
- ii) The mortality problem.
- iii) The mortal matrix problem.
- iv) The post Correspondence problem [pcp]

i) The halting problem:

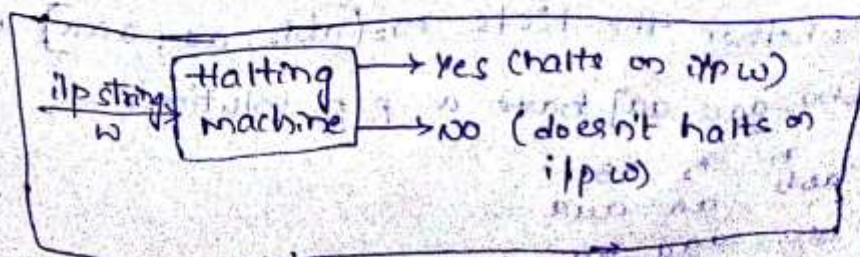
The halting problem inp: a Turing machine and the inp string w .

problem: Does the Turing machine finish computing of the string w in a finite no. of steps? The answer must be either YES (or) NO.

Proof:- At first, we will assume that a Turing machine exists to solve the problem. We will show and then it is contradicting itself.

We will call this Turing machine as a halting machine that produces a YES (or) NO in a finite amount of time.

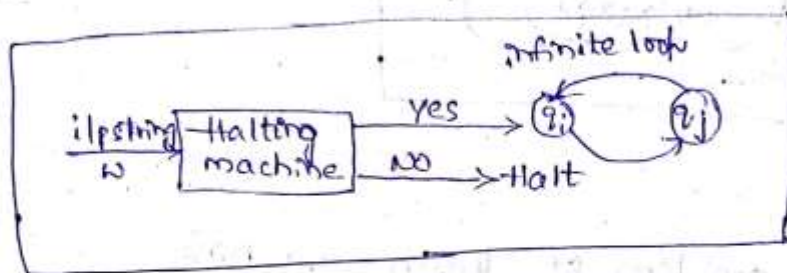
If the halting machine finishes in a finite amount of time then the inp comes as YES. otherwise, as NO.



Now, we will design an inverted halting machine as

- i) If HM returns YES then loop forever.
- ii) If HM returns NO then halt.

The following block diagram shows the inverted halting machine.



Further, a machine "HM" which ip itself is constructed as follows.

- i) IF HM halts on ip loop forever.
- ii) Else Halt

∴ Here, we have got a Contradiction. Hence, the halting problem is undecidable.

* Post Correspondence problem (PCP):—

- It was introduced by "Emil post" in 1946 is an undecidable decision problem.

The PCP problem over an ip alphabet Σ is stated as follows.

Given the following two lists, M and N, of non-empty strings over Σ

$$M = (x_1, x_2, x_3, \dots, x_n)$$

$$N = (y_1, y_2, y_3, \dots, y_n)$$

We can say that there is a PCP solution if for some $i_1, i_2, i_3, \dots, i_k$ where $1 \leq i_k \leq n$, the condition

$$x_{i_1} x_{i_2} x_{i_3} \dots x_{i_k} = y_{i_1} y_{i_2} y_{i_3} \dots y_{i_k} \text{ satisfies}$$

Ex:— Find whether the lists $M = [abb, aa, aaa]$ and $N = [bba, aaa, aa]$ have a PCP solution.

Sol:—

M)	x_1	x_2	x_3
	abb	aa	aaa
N)	y_1	y_2	y_3
	bba	aaa	aa

Here $x_2, x_1, x_3 = aaabbaaa$

$y_2, y_1, y_3 = aaabbaaa$

we can see that $x_2, x_1, x_3 = y_2, y_1, y_3$

Hence, the solution is $i=2, j=1, k=3$.

2) find whether the list $M = [ab, bab, bbaaa]$ and $N = [a, ba, bab]$ have a pcp solution.

Sol:-

	x_1	x_2	x_3
M	ab	bab	bbaaa
N	a	ba	bab

In this case, there is no solution because

$|x_2, x_1, x_3| \neq |y_2, y_1, y_3|$ lengths are not same.

Hence, it can be said that this pcp is an undecidable problem.

Modified Post Correspondence Problem:-

Given two lists $M = x_1, x_2, x_3, \dots, x_n$ and $N = y_1, y_2, y_3, \dots, y_n$.

Given a set of pairs of strings $\{(x_1, y_1), (x_2, y_2), \dots, (x_n, y_n)\}$

then the solution is an instance such that,

$$x_1 x_i x_{i_2} \dots x_{i_n} = y_1 y_i y_{i_2} \dots y_{i_n}$$

that means the pair (x_i, y_i) is forced to be at the beginning of the strings.

Ex:-

M	x_1	x_2	x_3
	11	100	111
N	111	001	11

Sol: \therefore Then the solution is $x_1, x_2, x_3 = y_1, y_2, y_3$.

$$x_1, x_2, x_3 = 11100111$$

$$y_1, y_2, y_3 = 11100111$$

That means it is essential to have x_i, y_i at the beginning of list.

Pard Np classes :-

- * P-problems
- * NP-problems
- * P vs NP

• P-problems:-

- * P is the class of problems that can be solved by deterministic algorithm in a polynomial type time $p(n^k)$ where n is the size of input string.
- * P-problem consist of a language accepted by deterministic Turing machine that runs in polynomial amount of time.

Ex:- 1) shortest path problem

2) Equivalence of NFA and DFA.

3) shortest cycle in a graph.

4) sorting algorithms

• NP-problems:-

- * NP-problem is a class of problems that can be solved by non-deterministic algorithms in a polynomial time $p(n^k)$ where n is the size of input string.

* NP-problems consists of a language accepted by non-deterministic Turing machine that runs in a polynomial amount of time.

Ex:- 1) Travelling sales man problem.

2) subgraph isomorphism

NP-problem classified into two types.

i) NP-hard problem.

ii) NP-complete problem.

• NP-hard problem:-

If there is a language x such that every language y in NP can be polynomially reducible to x , and we cannot prove that x is in NP, then x is said to be NP-hard problem.

Ex:- Turing machine halting problem.

• NP-complete problem:-

If there is a language x such that every language

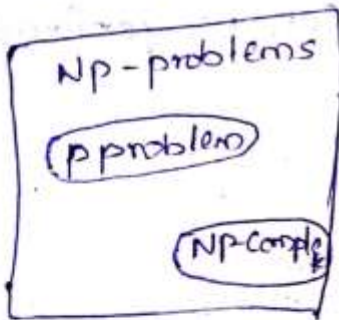
y in NP can be polynomially reducible to x and we can prove that x is in NP then x is said to be NP-complete problems.

- Ex: - 1) Travelling salesman problem.
2) Subgraph isomorphism.

P vs NP :-

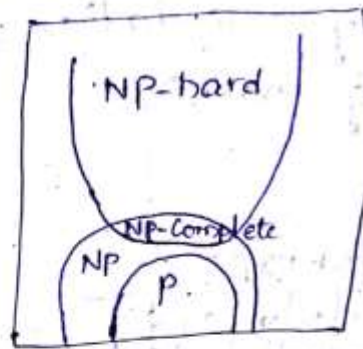
1. Kadner's theorem:

(a) $P \neq NP$



2. Euler's theorem:

(a) $P \neq NP$



(b) $P = NP$

