# FLUID MECHANICS AND HYDRAULIC MACHINES

*by*

**Dr. Pritam Kumar Das**

**Professor**

**Dept of Mechanical Engineering**

**Aditya Engineering College (A)**

# PROGRAM OUTCOMES (POs):

**After successful completion of the program, the graduates will be able to**

| | |
|---|---|
| PO 1 | Apply Knowledge of mathematics, science, engineering fundamentals and an engineering specialization to the solution of complex engineering problems. |
| PO 2 | Identify, formulate, research literature and analyze complex engineering problems, reaching substantiated conclusions using first principles of mathematics, natural sciences and engineering sciences. |
| PO 3 | Design solutions for complex engineering problems and design systems, components or processes that meet specified needs with appropriate consideration for public health and safety, cultural, societal, and environmental considerations. |
| PO 4 | Conduct investigations of complex problems using research-based knowledge and research methods including design of experiments, analysis and interpretation of data, and synthesis of information to provide valid conclusions. |
| PO 5 | Create, select and apply appropriate techniques, resources, and modern engineering and IT tools, including prediction and modelling, to complex engineering activities, with an understanding of the limitations. |
| PO 6 | Apply reasoning informed by contextual knowledge to assess societal, health, safety, legal and cultural issues and the consequent responsibilities relevant to professional engineering practice. |
| PO 7 | Understand the impact of professional engineering solutions in societal and environmental contexts and demonstrate knowledge of and need for sustainable development. |
| PO 8 | Apply ethical principles and commit to professional ethics and responsibilities and norms of engineering practice. |
| PO 9 | Function effectively as an individual, and as a member or leader in diverse teams and in multidisciplinary settings. |
| PO 10 | Communicate effectively on complex engineering activities with the engineering community and with society at large, such as being able to comprehend and write effective reports and design documentation, make effective presentations, and give and receive clear instructions. |
| PO 11 | Demonstrate knowledge and understanding of engineering management principles and apply these to one’s own work, as a member and leader in a team and to manage projects in multidisciplinary environments. |
| PO 12 | Recognize the need for and have the preparation and ability to engage in independent and life-long learning in the broadest context of technological change. |

**Course Outcomes:**

At the end of the Course, Student will be able to:

CO1: Explain the fluid properties and pressure measuring devices.

CO2: Calculate the buoyancy forces on a submerged body and continuity equation for 1-D flow.

CO3: Apply the concept of momentum equation on various application.

CO4: Calculate the application of boundary layer theory for fluid flow.

CO5: Analyze the impact of jet on vanes.

CO6: Calculate the performance characteristics of types of turbines and pumps.

| Bloom's Taxonomy Knowledge Level | Knowledge Level Representation |
|---|---|
| Remember | L1 |
| Understand | L2 |
| Apply | L3 |
| Analyse | L4 |
| Evaluate | L5 |
| Create | L6 |

**Mapping of Course Outcomes with Program Outcomes:**

| CO/PO | PO 1 | PO 2 | PO 3 | PO 4 | PO 5 | PO 6 | PO 7 | PO 8 | PO 9 | PO 10 | PO11 | PO12 | PSO1 | PSO2 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| **CO1** | 3 | 2 | - | - | - | - | - | - | - | - | - | 1 | - | 3 |
| **CO2** | 2 | 3 | - | - | - | - | - | - | - | - | - | - | - | 3 |
| **CO3** | 2 | 3 | - | - | - | - | - | - | - | - | - | - | - | 3 |
| **CO4** | 2 | 3 | - | - | - | - | - | - | - | - | - | - | - | 3 |
| **CO5** | 2 | 3 | - | - | - | - | - | - | - | - | - | - | - | 3 |
| **CO6** | 2 | 3 | - | - | - | - | - | - | - | - | - | 1 | - | 3 |

**Text Books:**

**1.** Fluid Mechanics and Hydraulic Machines, R. K. Bansal, Laxmi Publications (P) Ltd., 10th Edition.
**2.** Fluid Mechanics, Frank M. White, McGraw - Hill series, 7th edition.

**Reference Books:**

**1.** Hydraulics and Fluid mechanics including Hydraulics machines by P.N. Modi & S.M. Seth, Rajsons publications Pvt. Ltd. Standard Book House (2017 edition).
**2.** Fluid Mechanics and Hydraulic Machines, K. L Kumar, S. Chand publications, 2008.
**3.** Fluid Mechanics and Machinery, D. Rama Durgaiah, New Age International, Revised Edition.
**4.** Fluid Mechanics and Fluid Power Engineering, D. S. Kumar, Kotaria & Sons, Revised Edition.

**Web Links:**

**1.** http://nptel.ac.in/courses/112105171/1.
**2.** https://freevideolectures.com/course/89/fluid-mechanics.
**3.** https://nptel.ac.in/courses/112/105/112105183/.
**4.** https://ocw.mit.edu/courses.

# UNIT 1

# Introduction to Fluid Mechanics

# Lecture Plan

- **Introduction**
  - Definitions of: Solid, Fluid, Mechanics, Stresses, Fluid Mechanics
- **Where you find Fluid-Dynamics?**
- **Dimensions and units**
- **Physical properties of fluids**
  - (Mass, Density, Specific weight, Specific volume, Specific gravity, Viscosity, Surface tension, Vapor pressure) and their influence on fluid motion
- **Atmospheric pressure, Gauge pressure and Vacuum pressure**
- **Measurement of pressure**

# INTRODUCTION

## What is Fluid Mechanics? = Fluid + Mechanics

### What is a Solid & Fluid?

- **Solid** can resist an applied shear by deforming its shape **or** Deforms a fixed amount or breaks completely when a stress is applied on it.
- **Fluid:** Deforms continuously under the influence of shear stress, no matter how small its shape **or** Substances with no strength **or** Deform when forces are applied, ex. water and gases

"A fluid, such as water or air, deforms continuously when acted on by shearing stresses of any magnitude."

### What is Mechanics?

- **Mechanics** deal with both *stationery* and *moving* boundaries under the influence of forces **or** The study of motion and the forces which cause (or prevent) the motion. It is divided into three types:

- **Kinematics (kinetics):** Description of motion: displacement, velocity and acceleration.
- **Statics:** The study of forces acting on the particles or bodies at rest. The branch of the **mechanics** that deals with bodies at rest is called ***statics.***
- **Dynamics:** The study of forces acting on the particles and bodies in motion or the branch that deals with bodies in motion is called ***dynamics.***

**Stress = Force / Area**

- **Shear stress / Tangential stress:**

The force acting parallel to the surface per unit area of the surface.

- **Normal stress:**

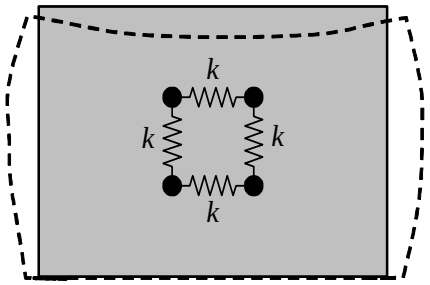
A force acting perpendicular to the surface per unit area of the surface.

❖ **Fluid Mechanics** deals with behavior of fluids at rest (*fluid statics*) or in motion (*fluid dynamics*) and the interaction of fluids with solids or other fluids at the boundaries **or** **Fluid mechanics** is a division in applied mechanics related to the behaviour of liquid or gas which is either in rest or in motion.

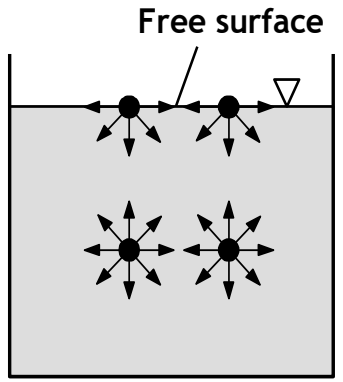
Fluids: Statics vs Dynamics



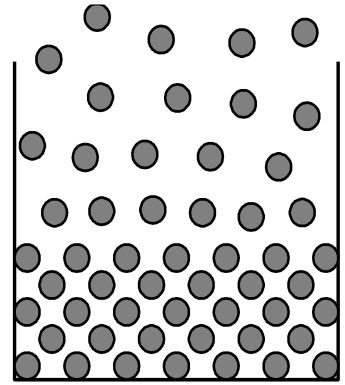
- For **solid**, imagine that the molecules can be fictitiously linked to each other with springs.
- In **fluid**, the molecules can move freely but are constrained through a traction force called **cohesion**. This force is interchangeable from one molecule to another.
- For **gases**, it is very weak which enables the gas to disintegrate and move away from its container.
- For **liquids**, it is stronger which is sufficient enough to hold the molecule together and can withstand high compression, which is suitable for application as hydraulic fluid such as oil.




(a) Solid

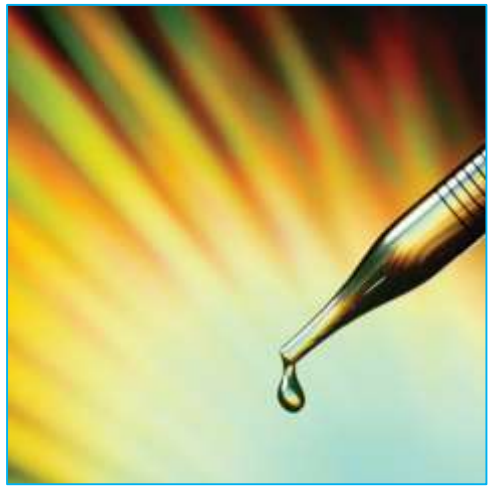



(b) Liquid



(c) Gas

*A drop forms when liquid is forced out of a small tube. The shape of the drop is determined by a balance of* pressure, gravity, and surface tension forces.






**Mechanics** of fluids is extremely important in many areas of engineering and science. Examples are:

**Biomechanics**:

- Blood flow through arteries and veins
- Flow of cerebral fluid

**Meteorology and Ocean Engineering**:

- Movements of air currents and water currents

**Chemical Engineering**:

- Design of chemical processing equipment

**Mechanical Engineering**

- Design of pumps, turbines, air-conditioning equipment, pollution-control equipment, **etc.**

**Civil Engineering**

- Transport of river sediments
- Pollution of air and water
- Design of piping systems
- Flood control systems

# Fluids omnipresent:

*Weather & climate*
*Vehicles: automobiles, trains, ships, and planes, etc.*
*Environment*
*Physiology and medicine*
*Sports & recreation*
*Many other examples!*

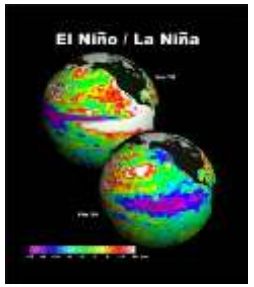
Tornadoes



Thunderstorm



Global Climate




Hurricanes



Aircraft



Surface ships



High-speed rail



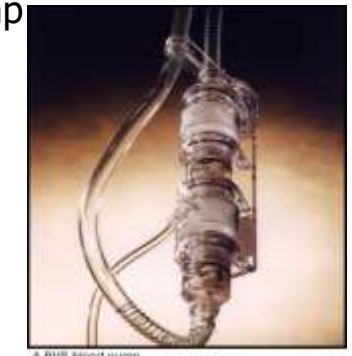
Submarines



Air pollution



Blood pump



Water sports



Cycling



Offshore racing




Ventricular assist device

River hydraulics





Auto racing






Surfing

# Interfacial Fluid Dynamics

- Frequently encounter
- Many complex phenomenon
  - ✓ Surface tension
  - ✓ Thermo-capillary flow
  - ✓ In industries: oil/gas
  - ✓ Hydrophobic nature

**Challenges :**

- Interfacial boundary condition.
- Numerical study becomes computationally very expensive.

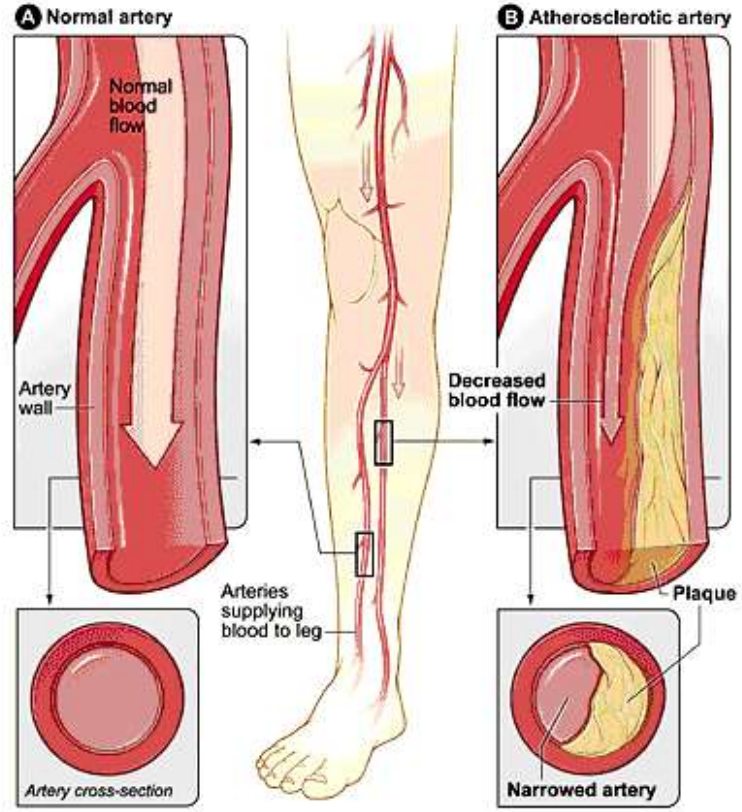




# Challenges in Fluid Mechanics

- Very Complex
- Rheology of blood
- Walls are flexible
- Pressure-wave travels along the arteries.
- Frequently encounter
- Bifurcation
- There are vary small veins

Blood Flow



# How Do We Study Fluid Mechanics?

**Basic laws of physics:**

- ❑Conservation of mass
- ❑Conservation of momentum – Newton's second law of motion
- ❑Conservation of energy: First law of thermodynamics
- ❑Second law of thermodynamics

**+ Equation of state**

Fluid properties e.g., density as a function of pressure and temperature.

**+ Constitutive laws**

Relationship between the stresses and the deformation of the material.

# Units and Dimensions

- A dimension is the measure by which a physical variable is expressed quantitatively and the unit is a particular way of attaching a number to the quantities of dimension.
- All the properties of fluid are assigned with certain unit and dimension. Some basic dimensions such as mass (M), length (L), time (t) and temperature (T) are selected as Primary / Fundamental dimensions / unit.

| Quantity | Symbol | Dimensions |
|---|---|---|
| Density | $r$ | $ML^{-3}$ |
| Specific Weight | $g$ | $ML^{-2}T^{-2}$ |
| Dynamic viscosity | $m$ | $ML^{-1}T^{-1}$ |
| Kinematic viscosity | $\nu$ | $L^{2}T^{-1}$ |
| Surface tension | $\sigma$ | $MT^{-2}$ |
| Bulk mod of elasticity | $E$ | $ML^{-1}T^{-2}$ |

**SI - International System**

| | |
|---|---|
| Temp | $^0K = {}^0C + 273.15$ |
| Force | Newton = N = 1 kg m / $s^2$ |
| Gravity | 9.81 m/$s^2$ |
| Work = Fxd | Joule = Nt-m |
| Power = F/t | Watt = Joule/sec |

| Quantity | SI Unit | English Unit |
|---|---|---|
| Length (L) | Meter (*m*) | Foot (*ft*) |
| Mass (m) | Kilogram (*kg*) | Slug (slug) = *lb*sec²/ft* |
| Time (t) | Second (*s*) | Second (*sec*) |
| Temperature (T) | Celcius ($^oC$) | Farenheit ($^oF$) |
| Force | Newton *(N)=kg*m/s²* | Pound (lb) |

| Quantity | Symbol | Dimensions |
|---|---|---|
| Velocity | $V$ | $LT^{-1}$ |
| Acceleration | $a$ | $LT^{-2}$ |
| Area | $A$ | $L^2$ |
| Volume | $\forall$ | $L^3$ |
| Discharge | $Q$ | $L^3T^{-1}$ |
| Pressure | $p$ | $ML^{-1}T^{-2}$ |
| Gravity | $g$ | $LT^{-2}$ |
| Temperature | $T'$ | $\Theta$ |
| Mass concentration | $C$ | $ML^{-3}$ |

| Quantity | Unit | Dimension |
|---|---|---|
| Temperature (T) | Kelvin | $M^0L^0T^0K^1$ |
| Heat (Q) | Joule | $[ML^2T^{-2}]$ |
| Specific Heat (*c*) | Joule/Kg–K | $[M^0L^2T^{-2}K^{-1}]$ |
| Thermal capacity | Joule/K | $[M^1L^2T^{-2}K^{-1}]$ |
| Latent heat (L) | Joule/kg | $[M^0L^2T^{-2}]$ |
| Gas constant (R) | Joule/mol-K | $[M^1L^2T^{-2}mol^{-1}K^{-1}]$ |
| Boltzmann constant (*k*) | Joule/K | $[M^1L^2T^{-2}K^{-1}]$ |
| Coefficient of thermal conductivity (K) | Joule/M-s-K | $[M^1L^1T^{-3}K^{-1}]$ |
| Stefan's constant ($\sigma$) | Watt/$m^2$–$K^4$ | $[M^1L^0T^{-3}K^{-4}]$ |
| Wien's constant (*b*) | Meter K | $[M^0L^1T^0K^1]$ |
| Planck's constant (*h*) | Joule s | $[M^1L^2T^{-1}]$ |
| Coefficient of Linear Expansion | $Kelvin^{-1}$ | $[M^0L^0T^0K^{-1}]$ |
| Mechanical eq. of Heat (J) | Joule/Calorie | $[M^0L^0T^0]$ |
| Vander wall's constant (*a*) | Newton $m^4$ | $[M^1L^5T^{-2}]$ |
| Vander wall's consatnt (*b*) | $m^3$ | $[M^0L^3T^0]$ |

## Derived Units(Named units derived from SI base units)

| Name | Symbol | Quantity | Equivalents | SI base unit |
|---|---|---|---|---|
| hertz | Hz | frequency | 1/s | s−1 |
| radian | rad | angle | m/m | dimensionless |
| steradian | sr | solid angle | m2/m2 | dimensionless |
| newton | N | force, weight | kg·m/s2 | kg·m·s−2 |
| pascal | Pa | pressure, stress | N/m2 | kg·m−1·s−2 |
| joule | J | energy, work, heat | N·m | kg·m2·s−2 |
| watt | W | power, radiant flux | J/s | kg·m2·s−3 |
| coulomb | C | electric charge or quantity of electricity | s·A | s·A |
| volt | V | voltage, electrical potential difference, electromotive force | W/A | kg·m2·s−3·A−1 |
| farad | F | electrical capacitance | C/V | kg−1·m−2·s4·A2 |
| ohm | Ω | electrical resistance, impedance, reactance | V/A | kg·m2·s−3·A−2 |
| siemens | S | electrical conductance | 1/Ω | kg−1·m−2·s3·A2 |
| weber | Wb | magnetic flux | J/A | kg·m2·s−2·A−1 |
| tesla | T | magnetic field strength, magnetic flux density | V·s/m2 | kg·s−2·A−1 |
| henry | H | Inductance | V·s/A | kg·m2·s−2·A−2 |

# Properties of Fluids

## 1. Density

- An *intensive* property of a substance
- May vary with temperature or pressure
- A *homogenous* material has the same density throughout
- For a substance of mass m and volume *V*

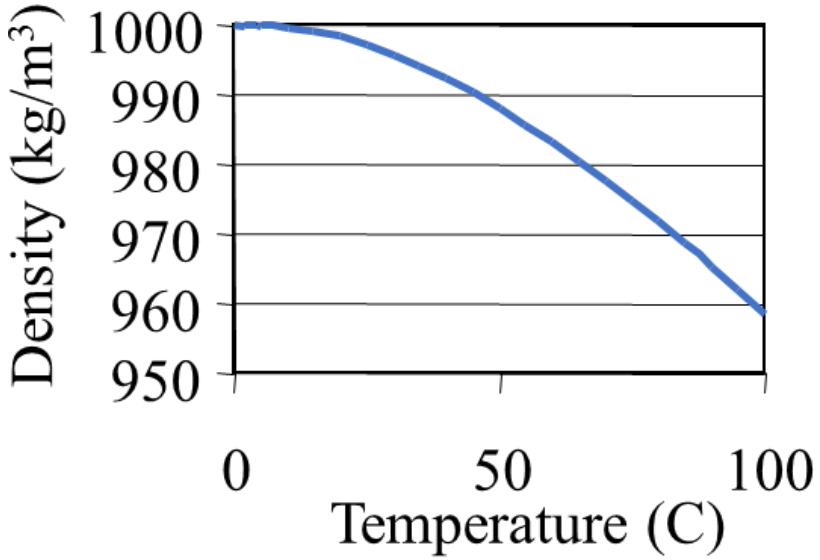
$$\rho = m/V$$

where the Greek letter $\rho$ (rho) is density

**Units:** $kg/m^3$

**Typical values:**

Water = 1000 $kg/m^3$; Air = 1.23 $kg/m^3$




## 2. Specific Weight or Weight Density w or γ

- Definition: weight of the fluid per unit volume of fluid
- Arising from the existence of a gravitational force
- The relationship $\gamma$ and g can be found using the following:

Since $\rho = m/\forall$

therefore $\gamma = \rho g$

**Units:** $N/m^3$

**Typical values:**

Water = 9814 $N/m^3$; Air = 12.07 $N/m^3$

$$w = \frac{\text{Weight of fluid}}{\text{Volume of fluid}} = \frac{(\text{Mass of fluid}) \times \text{Acceleration due to gravity}}{\text{Volume of fluid}}$$

$$= \frac{\text{Mass of fluid} \times g}{\text{Volume of fluid}}$$

$$= \rho \times g \qquad \left\{\because \frac{\text{Mass of fluid}}{\text{Volume of fluid}} = \rho\right\}$$

$$w = \rho g \qquad \ldots(1.1)$$

# Specific Gravity

The specific gravity (or relative density) can be defined in two ways:

**Definition 1**: A ratio of the density of a liquid to the density of water at standard temperature and pressure (STP) (20°C, 1 atm),

**Definition 2**: A ratio of the specific weight of a liquid to the specific weight of water at standard temperature and pressure (STP) (20°C, 1 atm),

**Unit:** dimensionless

$$SG = \frac{\rho_{liquid}}{\rho_{water@STP}} = \frac{\gamma_{liquid}}{\gamma_{water@STP}}$$

Mathematically, $S$(for liquids) = $\dfrac{\text{Weight density (density) of liquid}}{\text{Weight density (density) of water}}$

$S$(for gases) = $\dfrac{\text{Weight density (density) of gas}}{\text{Weight density (density) of air}}$

Thus weight density of a liquid = $S \times$ Weight density of water

$= S \times 1000 \times 9.81$ N/m$^3$

The density of a liquid = $S \times$ Density of water

$= S \times 1000$ kg/m$^3$.

specific gravity of mercury is 13.6,

**SPECIFIC VOLUME:**

It is defined as the volume of a fluid occupied by a unit mass or volume per unit mass of a fluid is called so.

**Problem 1:** **Calculate the specific weight, density and specific gravity of 1 litre of liquid which weighs 7 N.**

**Solution.** Given :

$$\text{Volume} = 1 \text{ litre} = \frac{1}{1000} \text{ m}^3 \quad \left( \because \quad 1 \text{ litre} = \frac{1}{1000} \text{ m}^3 \text{ or } 1 \text{ litre} = 1000 \text{ cm}^3 \right)$$

$$\text{Weight} = 7 \text{ N}$$

(*i*) Specific weight ($w$) $= \dfrac{\text{Weight}}{\text{Volume}} = \dfrac{7 \text{ N}}{\left(\frac{1}{1000}\right) \text{m}^3} =$ **7000 N/m$^3$. Ans.**

(*ii*) Density ($\rho$) $= \dfrac{w}{g} = \dfrac{7000}{9.81}$ kg/m$^3$ = **713.5 kg/m$^3$. Ans.**

(*iii*) Specific gravity $= \dfrac{\text{Density of liquid}}{\text{Density of water}} = \dfrac{713.5}{1000}$ $\{ \because$ Density of water = 1000 kg/m$^3 \}$

= **0.7135. Ans.**

## Problem 2:

*Calculate the density, specific weight and weight of one litre of petrol of specific gravity = 0.7*

**Solution.** Given : Volume = 1 litre = $1 \times 1000\ cm^3 = \frac{1000}{10^6}\ m^3 = 0.001\ m^3$

Sp. gravity $S = 0.7$

(*i*) *Density* ($\rho$)

Using equation (1.1*A*),

*Density* ($\rho$) $= S \times 1000\ kg/m^3 = 0.7 \times 1000 = \mathbf{700\ kg/m^3}$. **Ans.**

(*ii*) *Specific weight* (*w*)

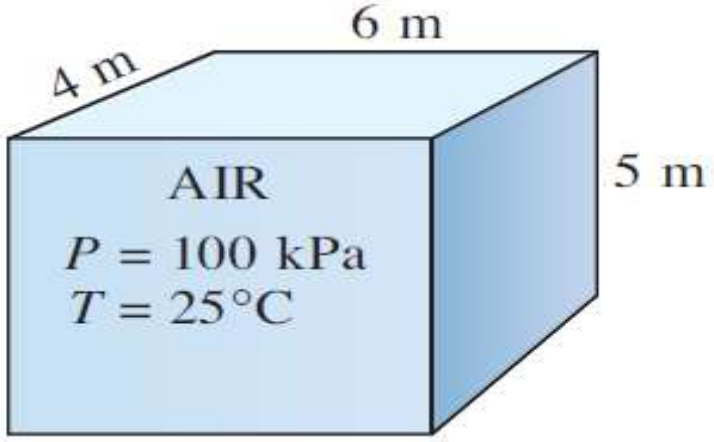
Using equation (1.1), $w = \rho \times g = 700 \times 9.81\ N/m^3 = \mathbf{6867\ N/m^3}$. **Ans.**

(*iii*) *Weight* (*W*)

We know that specific weight $= \frac{\text{Weight}}{\text{Volume}}$

or $w = \frac{W}{0.001}$ or $6867 = \frac{W}{0.001}$

$\therefore$ $W = 6867 \times 0.001 = \mathbf{6.867\ N}$. **Ans.**




**Problem 3:** A reservoir of oil has a mass of 825 kg. The reservoir has a volume of 0.917 $m^3$. Compute the density, specific weight, and specific gravity of the oil.

$$\rho_{oil} = \frac{mass}{volume} = \frac{m}{\forall} = \frac{825}{0.917} = 900 kg/m^3$$

$$\gamma_{oil} = \frac{\text{weight}}{\text{volume}} = \frac{mg}{\forall} = \rho g = 900 \text{x} 9.81 = 8829 \text{N/m}^3$$

$$SG_{oil} = \frac{\rho_{oil}}{\rho_{w@STP}} = \frac{900}{998} = 0.9$$

## Problem 4: Density, Specific Gravity, and Mass of Air in a Room

Determine the density, specific gravity, and mass of the air in a room whose dimensions are 4 m × 5 m × 6 m at 100 kPa and 25°C (Fig. 2–6).

**SOLUTION** The density, specific gravity, and mass of the air in a room are to be determined.

***Assumptions*** At specified conditions, air can be treated as an ideal gas.

***Properties*** The gas constant of air is $R = 0.287$ kPa·m³/kg·K.

***Analysis*** The density of the air is determined from the ideal-gas relation $P = \rho RT$ to be

$$\rho = \frac{P}{RT} = \frac{100 \text{ kPa}}{(0.287 \text{ kPa·m}^3/\text{kg·K})(25 + 273.15) \text{ K}} = \mathbf{1.17 \text{ kg/m}^3}$$

Then the specific gravity of the air becomes

$$\text{SG} = \frac{\rho}{\rho_{\text{H}_2\text{O}}} = \frac{1.17 \text{ kg/m}^3}{1000 \text{ kg/m}^3} = \mathbf{0.00117}$$

Finally, the volume and the mass of the air in the room are

$$V = (4 \text{ m})(5 \text{ m})(6 \text{ m}) = 120 \text{ m}^3$$

$$m = \rho V = (1.17 \text{ kg/m}^3)(120 \text{ m}^3) = \mathbf{140 \text{ kg}}$$

***Discussion*** Note that we converted the temperature to (absolute) unit K from (relative) unit °C before using it in the ideal-gas relation.

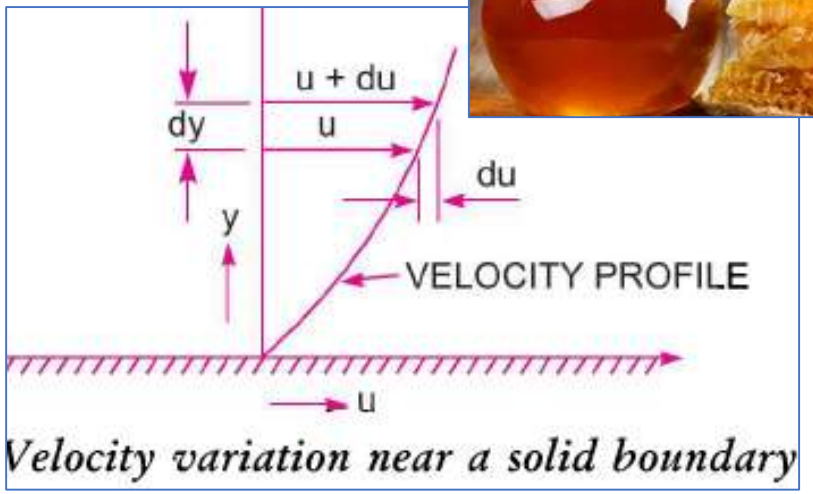
# Viscosity

- ✓ The viscosity ($\mu$) of a fluid is a measure of its resistance to deformation at a given rate. For liquids, it corresponds to the informal concept of "**thickness**": for example, syrup has a higher viscosity than water.
- ✓ Viscosity, is a measurement of resistance to fluid flow as a result of intermolecular cohesion.
- ✓ It defines as the resistance of a fluid which is being deformed by the application of shear stress **Or** Different fluids deform at different rates under the same shear stress. The ease with which a fluid pours is an indication of its viscosity.
- ✓ The viscosity is also known as dynamic viscosity. Examples of highly viscous fluids: **Honey Molasses, Tar, 20w-50 oil**
- ✓ When two layers of fluid, a distance dy, move one over another at different velocities, let u and u + du,. The top layer causes a shear stress on the adjacent layer while the lower layer causes a shear stress on the adjacent top layer.
- ✓ This shear stress is proportional to the rate of change of velocity with respect to y.
- ✓ It is denoted by symbol tau (τ).

$$\tau = \mu \frac{du}{dy}$$

τ = shear stress
μ = viscosity of fluid
**du/dy** = shear rate, rate of strain or velocity gradient





*Velocity variation near a solid boundary*

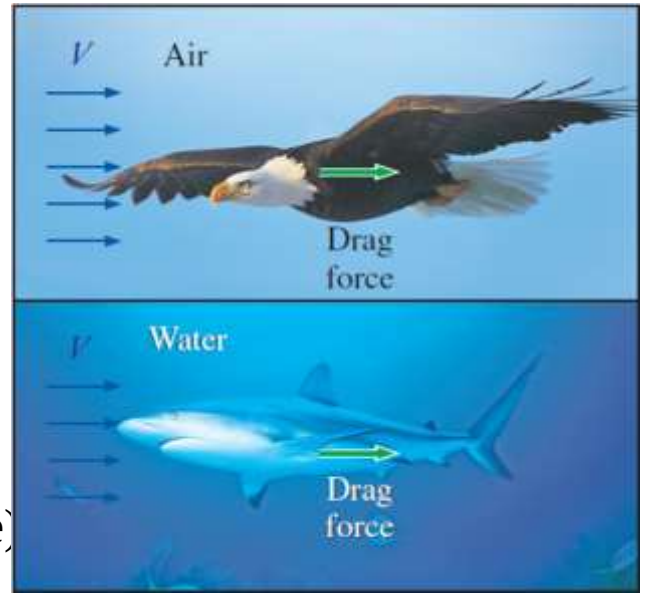
**Units:** SI unit of viscosity = $Ns/m^2$ = Pa s.

Typical values: Water = $1.14 \times 10^{-3}$ kg/m/s; Air = $1.78 \times 10^{-5}$ kg/m/s

The unit of viscosity in CGS is also called Poise which is equal to $\frac{\text{dyne-sec}}{\text{cm}^2}$

Ethanol
Water
Honey
Sludge
Slurry
Pastes

- ✓ Drag force: The force a flowing fluid exerts on a body in the flow direction. The magnitude of this force depends, in part, on viscosity
- ✓ A fluid moving relative to a body exerts a drag force on the body, partly because of friction caused by viscosity.






## ❑ Fundamental Mechanisms:

- ❖ Gases - transfer of molecular momentum
  - ✓ Viscosity increases as temperature increases.
  - ✓ Viscosity increases as pressure increases.
- ❖ Liquids - cohesion and momentum transfer
  - ✓ Viscosity decreases as temperature increases.
  - ✓ Relatively independent of pressure (incompressible)

- ➢ Common fluids, e.g., water, air, mercury obey Newton's law of viscosity and are known as **Newtonian fluid**.
- ➢ Other classes of fluids, e.g., paints, polymer solution, blood do not obey the typical linear relationship of stress and strain. They are known as **non-Newtonian fluids.**

# Newtonian and Non-Newtonian Fluid

Fluid —obey→ Newton's law of viscosity —refer→ Newtonian fluids

Example:
Air
Water
Oil
Gasoline
Alcohol
Kerosene
Benzene
Glycerin

***Newton's' law of viscosity is given by;***

$$\tau = \mu \frac{du}{dy} \qquad (1.1)$$

$\tau$ = shear stress
$\mu$ = viscosity of fluid
$du/dy$ = shear rate, rate of strain or velocity gradient

- The viscosity $\mu$ is a function only of the condition of the fluid, particularly its temperature.
- The magnitude of the velocity gradient ($du/dy$) has no effect on the magnitude of $\mu$.

Fluid —Do not obey→ Newton's law of viscosity → Non- Newtonian fluids

- The viscosity of the non-Newtonian fluid is dependent on the *velocity gradient* as well as the *condition of the fluid*.

***Newtonian Fluids***

- A linear relationship between shear stress and the velocity gradient (rate of shear),
- the slope is constant
- the viscosity is constant

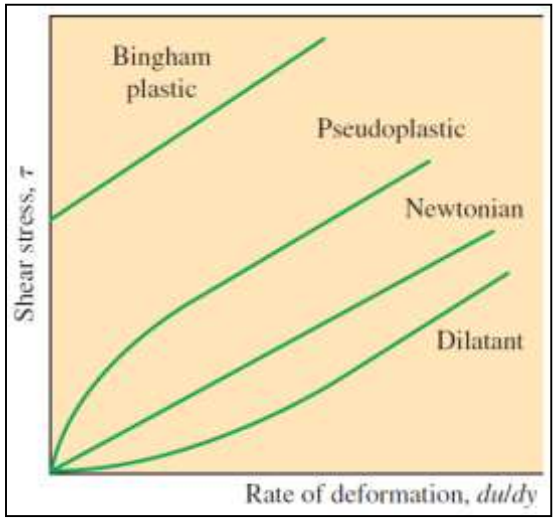
***Non-Newtonian fluids***

- slope of the curves for non-Newtonian fluids varies

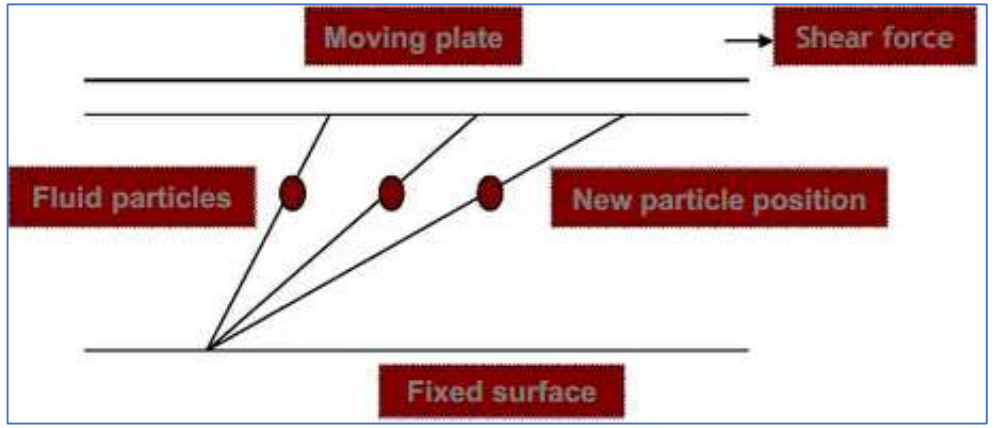
**Bingham plastic:** Resist a small shear stress but flow easily under large shear stresses, e.g. sewage sludge, toothpaste, and jellies.

**Pseudo plastic:** Most Non-Newtonian fluids fall under this group. Viscosity decreases with increasing velocity gradient, e.g. colloidal substances like clay, milk, and cement.

**Dilatants:** Viscosity decreases with increasing velocity gradient, e.g. quicksand.




If the gradient *m* is constant, the fluid is termed as ***Newtonian fluid***. Otherwise, it is known as ***non-Newtonian fluid***. Fig. shows several Newtonian and non-Newtonian fluids.




# Kinematic viscosity, $\nu$

Definition: is the ratio of the viscosity to the density;

$$\nu = \mu / \rho$$

will be found to be important in cases in which significant viscous and gravitational forces exist.

**Units:** $m^2/s$

**Typical values:**

Water = 1.14x10-6 $m^2/s$; Air = 1.46x10-5 $m^2/s$;

In general,

viscosity of liquids ↓ with ↑ temperature, whereas

viscosity of gases ↑ with ↑ in temperature.

# Types of Fluid:

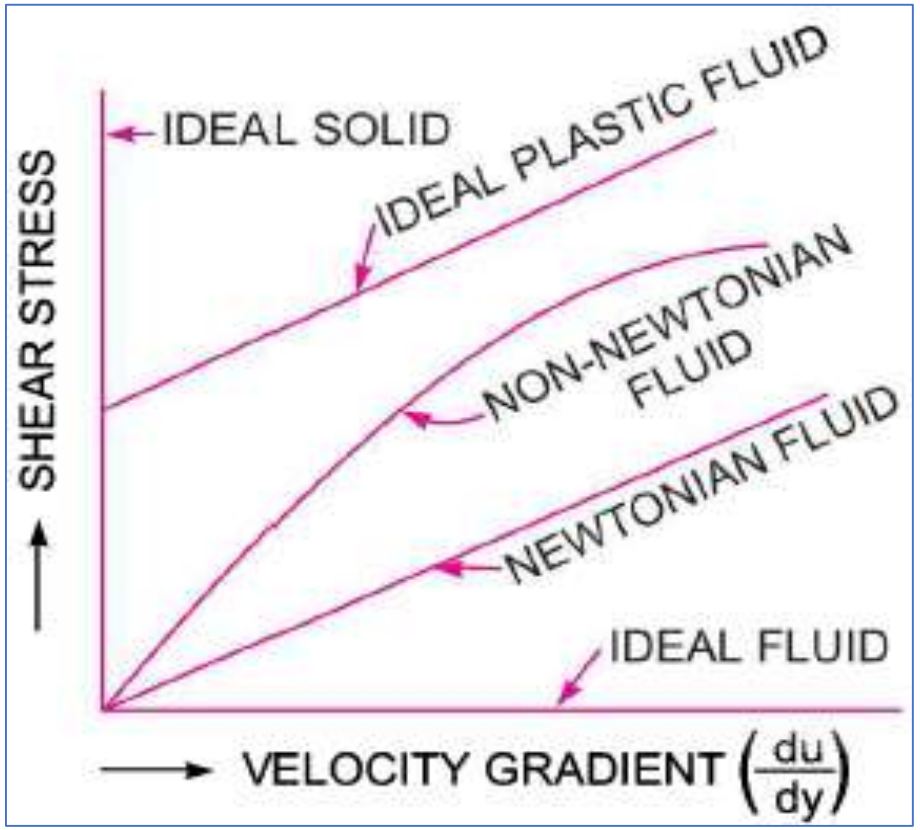
1. Ideal fluid 2. Real fluid 3. Newtonian fluid 4. Non-Newtonian fluid. 5. Ideal plastic fluid

**1. Ideal Fluid.** A fluid, which is incompressible and is having no viscosity, is known as an ideal fluid. Ideal fluid is only an imaginary fluid as all the fluids, which exist, have some viscosity.

**2. Real Fluid.** A fluid, which possesses viscosity, is known as real fluid. All the fluids, in actual practice, are real fluids.

**3. Newtonian Fluid.** A real fluid, in which the shear stress is directly proportional to the rate of shear strain (or velocity gradient), is known as a Newtonian fluid.

**4. Non-Newtonian Fluid.** A real fluid, in which the shear stress is not proportional to the rate of shear strain (or velocity gradient), known as a Non-Newtonian fluid.




**5. Ideal Plastic Fluid.** A fluid, in which shear stress is more than the yield value and shear stress is proportional to the rate of shear strain (or velocity gradient), is known as ideal plastic fluid.

**Problem 5:** *If the velocity distribution over a plate is given by* $u = \frac{2}{3} y - y^2$ *in which u is the velocity in metre per second at a distance y metre above the plate, determine the shear stress at y = 0 and y = 0.15 m. Take dynamic viscosity of fluid as 8.63 poises.*

**Solution.** Given : $u = \frac{2}{3} y - y^2 \quad \therefore \quad \frac{du}{dy} = \frac{2}{3} - 2y$

$$\left(\frac{du}{dy}\right)_{\text{at } y=0} \text{ or } \left(\frac{du}{dy}\right)_{y=0} = \frac{2}{3} - 2(0) = \frac{2}{3} = 0.667$$

Also $$\left(\frac{du}{dy}\right)_{\text{at } y=0.15} \text{ or } \left(\frac{du}{dy}\right)_{y=0.15} = \frac{2}{3} - 2 \times .15 = .667 - .30 = 0.367$$

Value of $\mu = 8.63$ poise $= \frac{8.63}{10}$ SI units $= 0.863$ N s/m$^2$

Now shear stress is given by equation (1.2) as $\tau = \mu \frac{du}{dy}$.

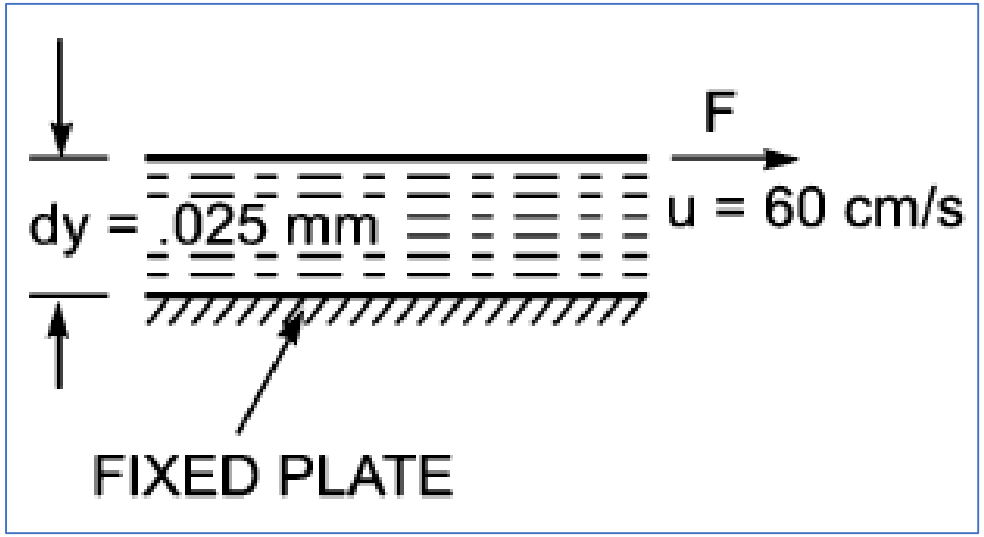
(*i*) Shear stress at $y = 0$ is given by

$$\tau_0 = \mu \left(\frac{du}{dy}\right)_{y=0} = 0.863 \times 0.667 = \mathbf{0.5756\ N/m^2.\ Ans.}$$

(*ii*) Shear stress at $y = 0.15$ m is given by

$$(\tau)_{y=0.15} = \mu \left(\frac{du}{dy}\right)_{y=0.15} = 0.863 \times 0.367 = \mathbf{0.3167\ N/m^2.\ Ans.}$$

**Problem 6:** A plate 0.025 mm distant from a fixed plate moves at 60 cm/s and requires a force of 2 N per meter square to maintain its speed. Determine fluid viscosity between the plates.




**Given**: distance between two plates dy = 0.025 mm

$= 0.025 \times 10^{-3}$ m

Velocity of upper plate u = 60 cm/s = 0.6 m/s

Force on upper plate, F = 2 ($N/m^2$) = Shear stress $\tau$

du = change of velocity = u – 0 = 0.6 m/s

dy = change of distance = $0.025 \times 10^{-3}$m

Viscosity, $$\mu = \frac{\tau}{du/dy} = \frac{2}{(0.6/0.25\times10^{-3})}$$

$8.33 \times 10^{-5}$ ($Ns/m^2$) = 8.33 × 10-5 Pas.

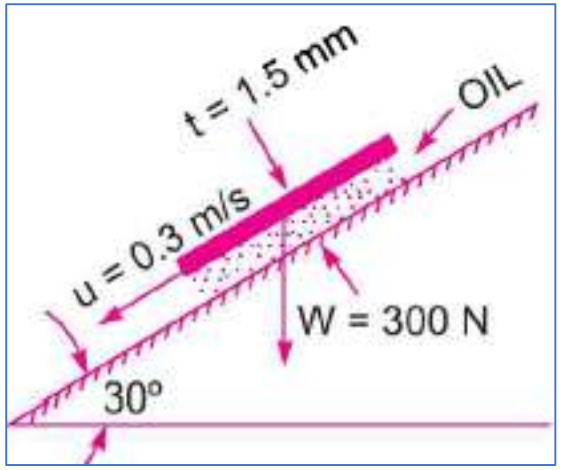
**Problem 7:** Calculate the dynamic viscosity of oil which is used for lubrication between a square plate of size 0.8 m x 0.8 m and an inclined angle of inclination 30 as shown in figure. The weight of the square plate is 300 N and it slides down the inclined plane with a uniform velocity of 0.3 m/s. The thickness of the oil film is 1.5 mm.

Area of plate, $A = 0.8 \times 0.8 = 0.64 \text{ m}^2$

Angle of plane, $\theta = 30°$

Weight of plate, $W = 300 \text{ N}$

Velocity of plate, $u = 0.3 \text{ m/s}$




Thickness of oil film, $t = dy = 1.5 \text{ mm} = 1.5 \times 10^{-3} \text{ m}$

Let the viscosity of fluid between plate and inclined plane is $\mu$.

Component of weight $W$, along the plane $= W \cos 60° = 300 \cos 60° = 150 \text{ N}$

Thus the shear force, $F$, on the bottom surface of the plate = 150 N

ınd shear stress, $$\tau = \frac{F}{\text{Area}} = \frac{150}{0.64} \text{ N/m}^2$$

Now using equation (1.2), we have

$$\tau = \mu \frac{du}{dy}$$

vhere $du$ = change of velocity $= u - 0 = u = 0.3$ m/s

$dy = t = 1.5 \times 10^{-3}$ m

$$\therefore \quad \frac{150}{0.64} = \mu \frac{0.3}{1.5 \times 10^{-3}}$$

$$\therefore \quad \mu = \frac{150 \times 1.5 \times 10^{-3}}{0.64 \times 0.3} = 1.17 \text{ N s/m}^2 = 1.17 \times 10 = \mathbf{11.7 \text{ poise}}$$

**Problem 8:** The velocity distribution for flow over a flat plate is given by $u=3/4\ y-y^2$ in which u is the velocity in meter per second at a distance y meter above the plate. Determine the shear stress at y=0.15m.Take dynamic viscosity of fluid as 8.5 poise.

$$u = \frac{3}{4}y - y^2$$

$$\frac{du}{dy} = \frac{3}{4} - 2y$$

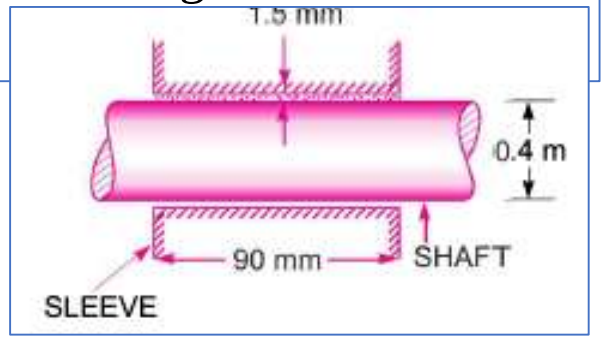
At $y = 0.15$,

$$\frac{du}{dy} = \frac{3}{4} - 2 \times 0.15 = 0.75 - 0.30 = 0.45$$

$$\mu = 8.5 \text{ poise} = \frac{8.5}{10}\frac{\text{N s}}{\text{m}^2} \quad \left(\because 10 \text{ poise} = 1\frac{\text{Ns}}{\text{m}^2}\right)$$

$$\tau = \mu \frac{du}{dy} = \frac{8.5}{10} \times 0.45 \frac{\text{N}}{\text{m}^2} = \mathbf{0.3825}\ \frac{\mathbf{N}}{\mathbf{m^2}}$$

**Problem 9:** The dynamic viscosity of an oil used for lubrication between a shaft and a sleeve is 6 Poise. The shaft is of diameter 0.4 m and rotates at 190 rpm. Calculate the power lost in the bearing for the sleeve length of 90 mm. The thickness of the film is 1.5 mm. (Hint: u =ЛDN/60, T = FD/2, Power = 2ЛNT/60)




Viscosity $\mu = 6$ poise

$$= \frac{6}{10}\frac{\text{N s}}{\text{m}^2} = 0.6\ \frac{\text{N s}}{\text{m}^2}$$

Dia. of shaft, $D = 0.4$ m

Speed of shaft, $N = 190$ r.p.m

Sleeve length, $L = 90 \text{ mm} = 90 \times 10^{-3}$ m

Thickness of oil film, $t = 1.5 \text{ mm} = 1.5 \times 10^{-3}$ m

Tangential velocity of shaft, $u = \frac{\pi DN}{60} = \frac{\pi \times 0.4 \times 190}{60} = 3.98$ m/s

Using the relation $\tau = \mu \frac{du}{dy}$

where $du$ = Change of velocity = $u - 0 = u = 3.98$ m/s

$dy$ = Change of distance = $t = 1.5 \times 10^{-3}$ m

$$\tau = 10 \times \frac{3.98}{1.5 \times 10^{-3}} = 1592 \text{ N/m}^2$$

This is shear stress on shaft

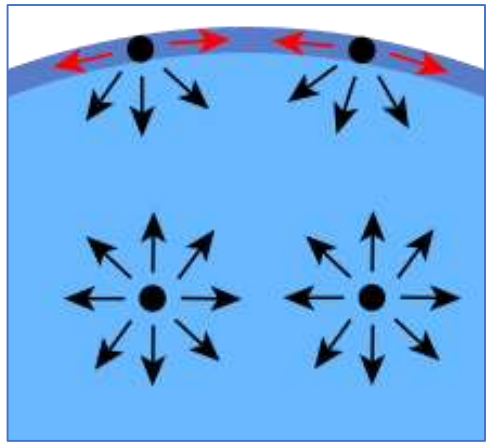
$\therefore$ Shear force on the shaft, $F$ = Shear stress × Area

$$= 1592 \times \pi D \times L = 1592 \times \pi \times .4 \times 90 \times 10^{-3} = 180.05 \text{ N}$$

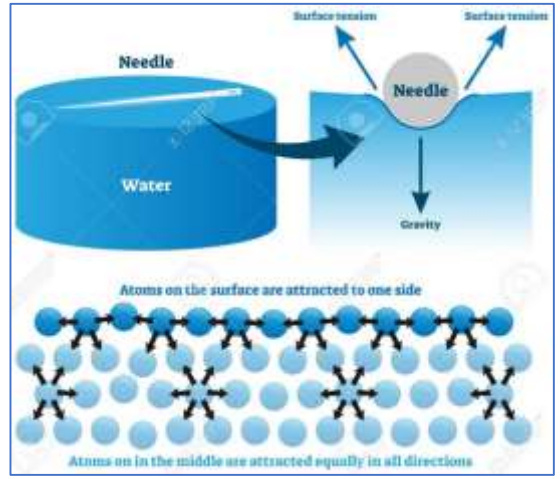
Torque on the shaft, $T = \text{Force} \times \frac{D}{2} = 180.05 \times \frac{0.4}{2} = 36.01$ Nm

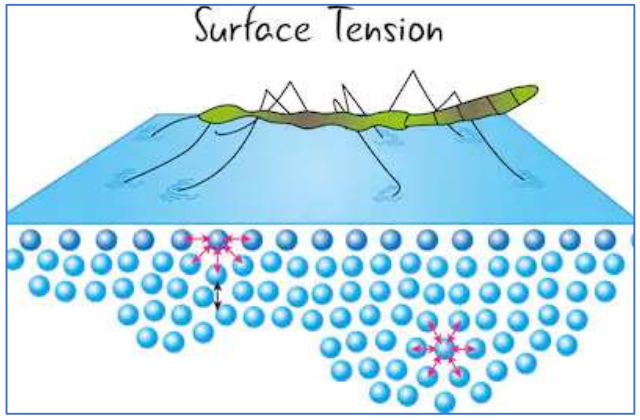
$\therefore$ *Power lost $= \frac{2\pi NT}{60} = \frac{2\pi \times 190 \times 36.01}{60} = \mathbf{716.48\ W.\ Ans.}$

# Surface Tension

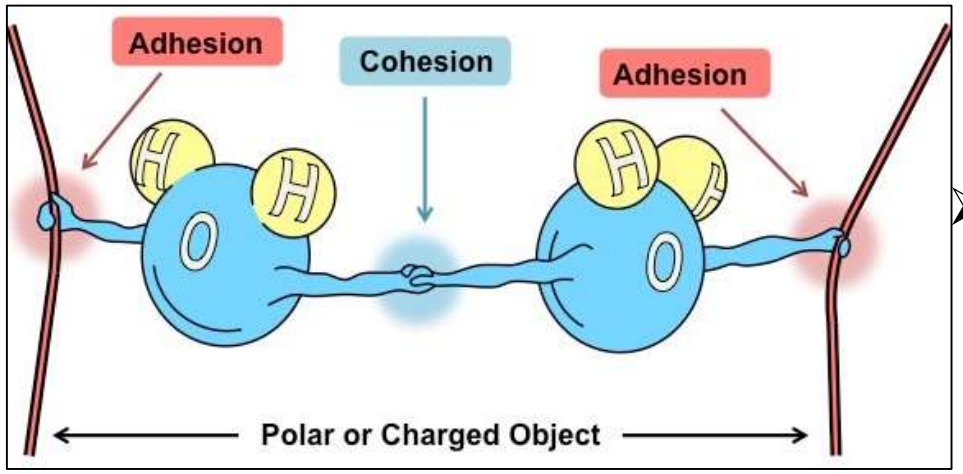


- **Surface tension ($\sigma$)** is the tendency of liquid surfaces to shrink into the minimum surface area possible **Or** It can be defined as the intensity of intermolecular traction per unit length along the free surface of a fluid, and its ***SI unit is N/m***.
- At liquid–air interfaces, $\sigma$ results from the greater attraction of liquid molecules to each other (due to cohesion) than to the molecules in the air (due to adhesion).
- The $\sigma$ effect is caused by unbalanced cohesion forces at fluid surfaces which produce a downward resultant force which can physically seen as a membrane. The property of the surface of a liquid that allows it to resist an external force, due to the cohesive nature of its molecules.
- The coefficient is inversely proportional to temperature and is also dependent on the type of the solid interface. For example, a drop of water on a glass surface will have a different coefficient from the similar amount of water on a wood surface.
- $\sigma$ allows insects (e.g. water striders), usually denser than water, to float and slide on a water surface.

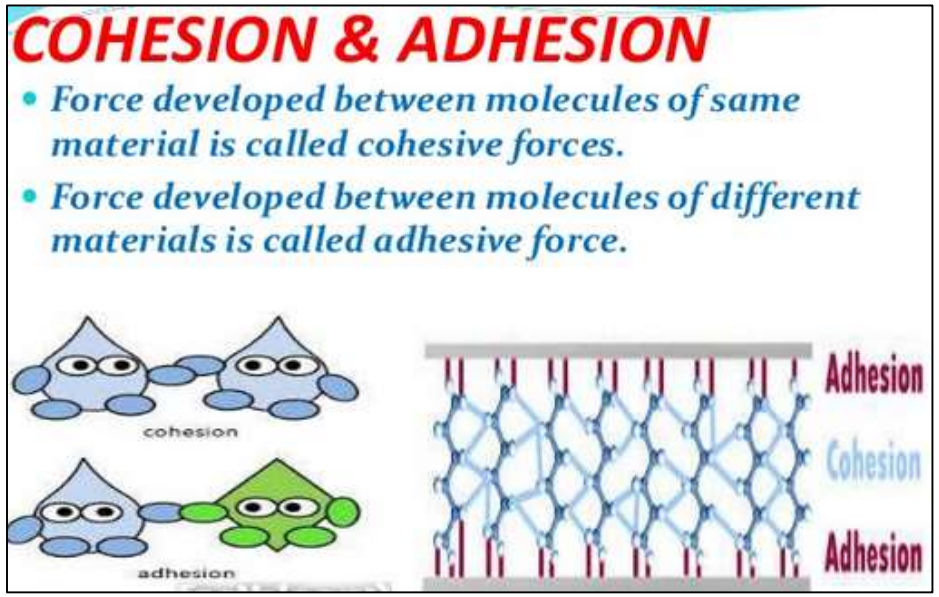


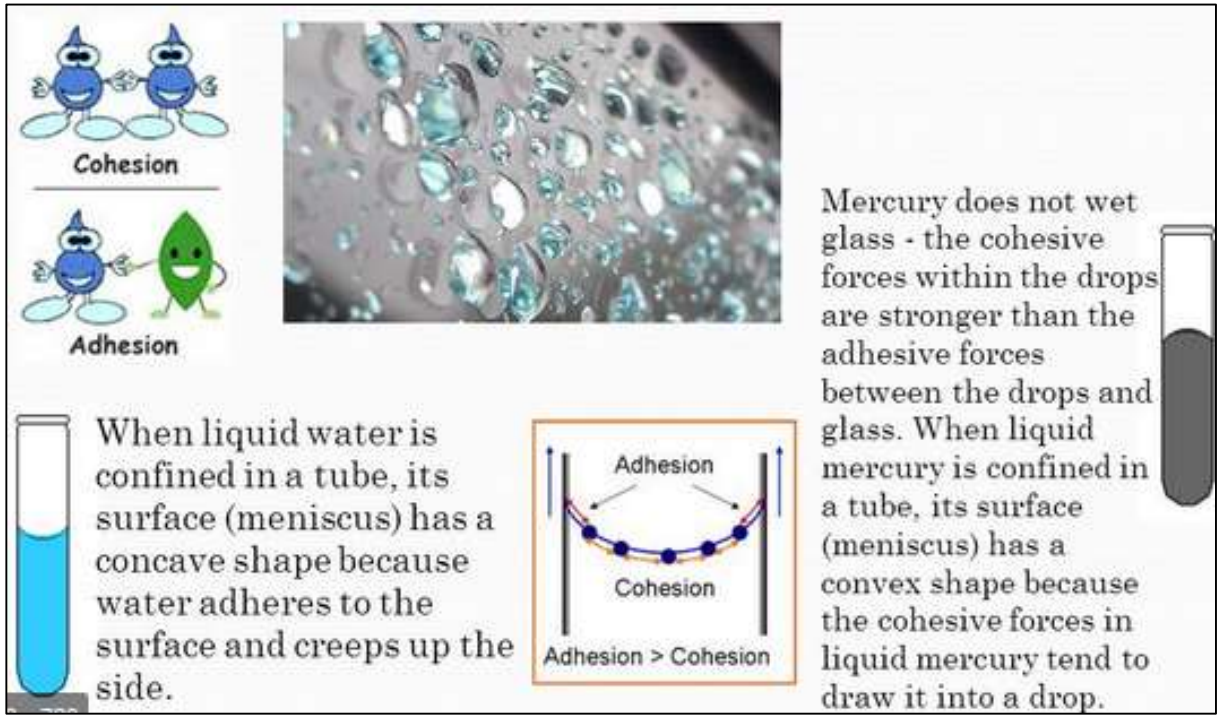



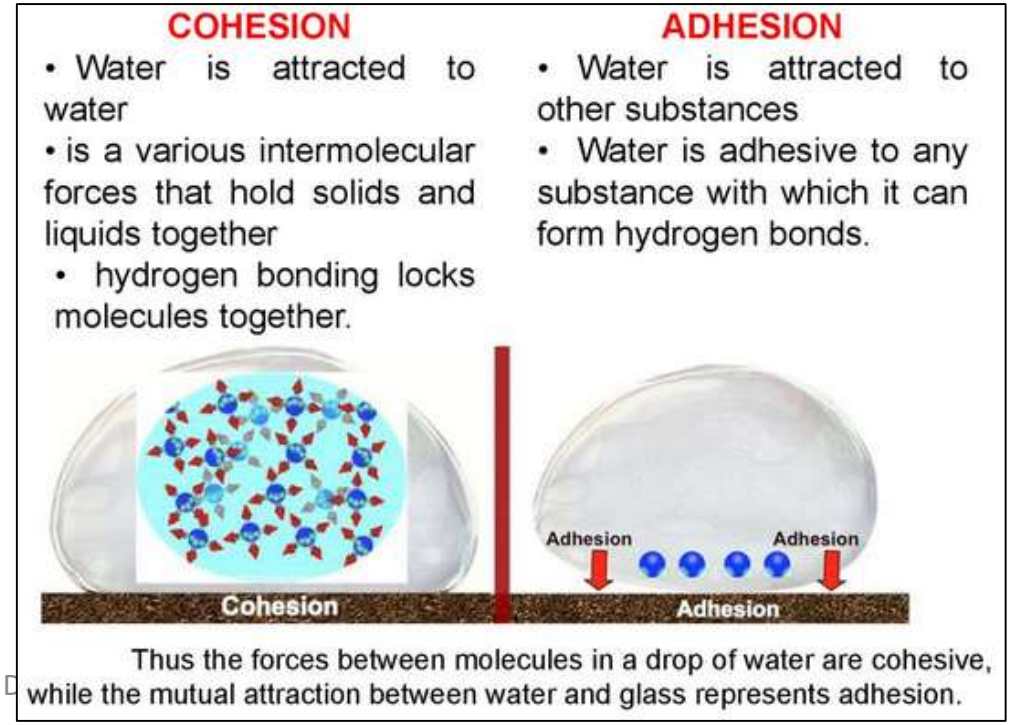

- ✓ Due to the cohesive forces a molecule is pulled equally in every direction by neighbouring liquid molecules, resulting in a net force of zero.
- ✓ The molecules at the surface do not have the same molecules on all sides of them and therefore are pulled inward. This creates some internal pressure and forces liquid surfaces to contract to the minimum area




- The force of cohesion is defined as the force of attraction between molecules of the same substance.
- The force of adhesion is defined as the force of attraction between different substances, such as glass and water.



Some important real life examples are

(i) Formation of water bubbles.

(ii) Formation of rain droplets.

(iii) Collection of dust particles on water surface.

(iv) A small needle can gently placed on the liquid surface without sinking.

(v) Breakup of liquid jets.

(vi) Capillary rise and capillary siphoning.

ST depends directly upon the intermolecular cohesion and hence the cohesion decreases with temperature rise, the ST also decreases with rise in temperature. It also depends upon the following factors.

(i) Nature of the liquid.

(ii) Nature of the surrounding liquid.

(iii) Kinetic energy of the liquid.

# ST on Liquid Droplets



Consider a small spherical droplet of a liquid of radius r.

Let $\sigma$ = Surface tension of the liquid

$p$ = Pressure intensity inside the droplet (in excess of the outside pressure intensity)

$d$ = Dia. of droplet.

Let the droplet is cut into two halves. The forces acting on one half (say left half) will be

(*i*) tensile force due to surface tension acting around the circumference of the cut portion

$= \sigma \times$ Circumference

$= \sigma \times \pi d$

(*ii*) pressure force on the area $\frac{\pi}{4} d^2 = p \times \frac{\pi}{4} d^2$ as shown in Fig. 1.11 (*c*). These two forces will be equal and opposite under equilibrium conditions, *i.e.*,

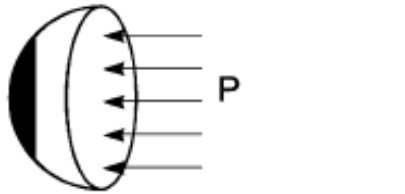


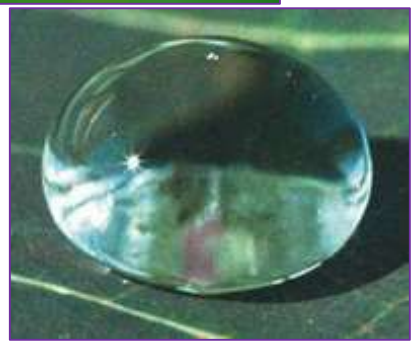

$$p \times \frac{\pi}{4} d^2 = \sigma \times \pi d$$

or

$$p = \frac{\sigma \times \pi d}{\frac{\pi}{4} \times d^2} = \frac{4\sigma}{d} \qquad ...(1.14)$$

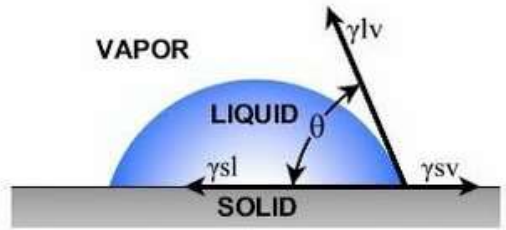
Equation (1.14) shows that with the decrease of diameter of the droplet, pressure intensity inside the droplet increases.



**Young's Equation**

$$\gamma^{sv} = \gamma^{sl} + \gamma^{lv}\cos\theta$$




$\theta$ is the contact angle

$\gamma^{sl}$ is the solid/liquid interfacial free energy

$\gamma^{sv}$ is the solid surface free energy

$\gamma^{lv}$ is the liquid surface free energy

# ST on a Hollow Bubble

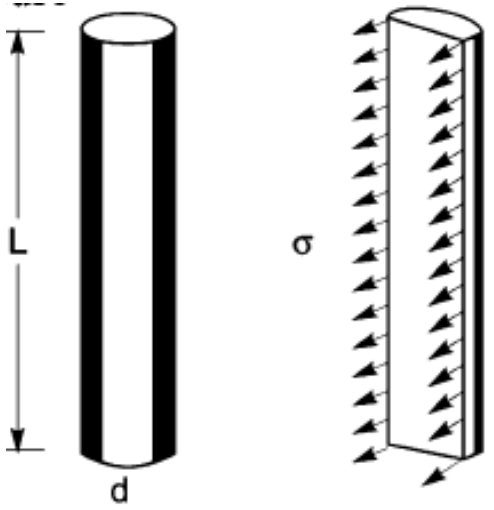


A hollow bubble like a soap bubble in air has two surfaces in contact with air, one inside and other outside. Thus two surfaces are subjected to surface tension. In such case, we have

$$p \times \frac{\pi}{4} d^2 = 2 \times (\sigma \times \pi d)$$

$$p = \frac{2\sigma\pi d}{\frac{\pi}{4} d^2} = \frac{8\sigma}{d}$$

# ST on a Liquid Jet






Let $p$ = Pressure intensity inside the liquid jet above the outside pressure

$\sigma$ = Surface tension of the liquid.

Consider the equilibrium of the semi jet, we have

Force due to pressure = $p \times$ area of semi jet

$= p \times L \times d$

Force due to surface tension $= \sigma \times 2L$.

Equating the forces, we have

$$p \times L \times d = \sigma \times 2L$$

$$\therefore \quad p = \frac{\sigma \times 2L}{L \times d}$$

**Problem 10:** The surface Tension of water is contact with air at 20°C is 0. 0725 N/m. The pressure inside a droplet of water is to be 0.02 N/cm$^2$ greater than the outside pressure. Calculate the diameter of the droplet of water.

Surface tension, $\sigma = 0.0725$ N/m

Pressure intensity, $p$ in excess of outside pressure is

$$p = 0.02 \text{ N/cm}^2 = 0.02 \times 10^4 \frac{\text{N}}{\text{m}^2}$$

Let $d$ = dia. of the droplet

we get $p = \frac{4\sigma}{d}$ or $0.02 \times 10^4 = \frac{4 \times 0.0725}{d}$

$$d = \frac{4 \times 0.0725}{0.02 \times (10)^4} = .00145 \text{ m} = .00145 \times 1000 = \mathbf{1.45 \text{ mm.}}$$

**Problem 11:** *Find the surface tension in a soap bubble of 40 mm diameter when the inside pressure is 2.5 N/m$^2$ above atmospheric pressure.*

**Solution.** Given :

Dia. of bubble, $d = 40 \text{ mm} = 40 \times 10^{-3}$ m

Pressure in excess of outside, $p = 2.5 \text{ N/m}^2$

For a soap bubble, using equation (1.15), we get

$$p = \frac{8\sigma}{d} \quad \text{or} \quad 2.5 = \frac{8 \times \sigma}{40 \times 10^{-3}}$$

$$\sigma = \frac{2.5 \times 40 \times 10^{-3}}{8} \text{ N/m} = \mathbf{0.0125 \text{ N/m. Ans.}}$$

**Problem 12:** *The pressure outside the droplet of water of diameter 0.04 mm is 10.32 N/cm$^2$ (atmospheric pressure). Calculate the pressure within the droplet if surface tension is given as 0.0725 N/m of water.*

**Solution.** Given :

Dia. of droplet, $d = 0.04 \text{ mm} = .04 \times 10^{-3} \text{ m}$

Pressure outside the droplet $= 10.32 \text{ N/cm}^2 = 10.32 \times 10^4 \text{ N/m}^2$

Surface tension, $\sigma = 0.0725 \text{ N/m}$

The pressure inside the droplet, in excess of outside pressure is given by equation (1.14)

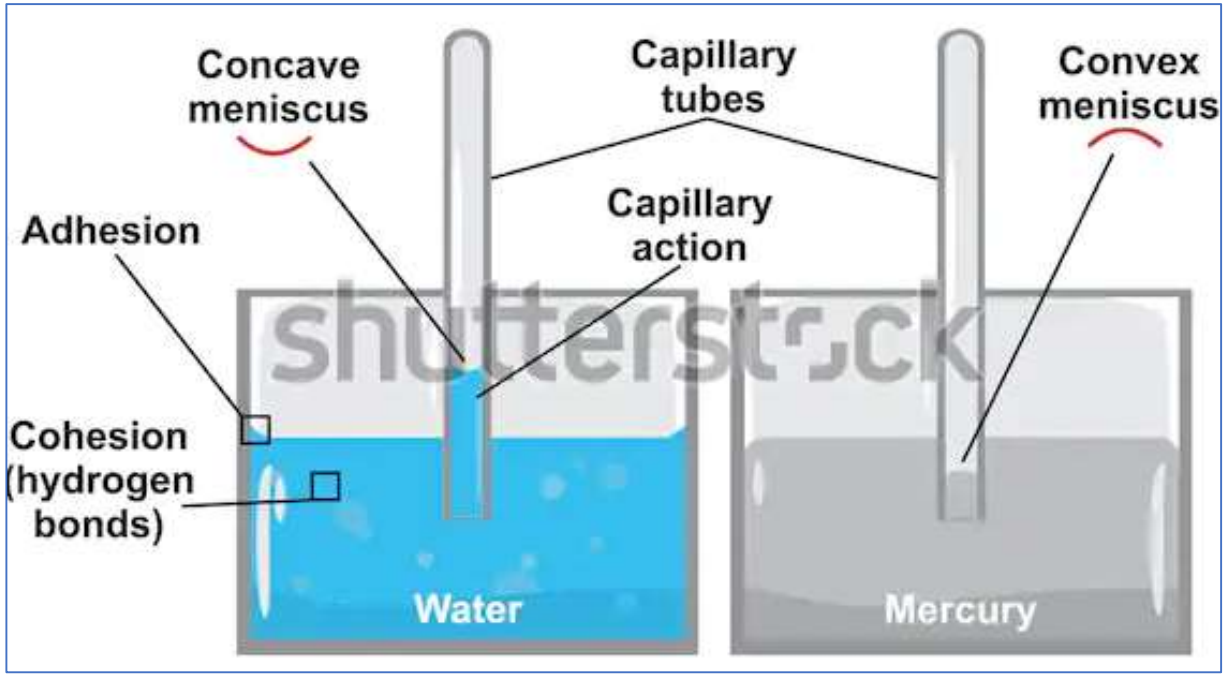
or
$$p = \frac{4\sigma}{d} = \frac{4 \times 0.0725}{.04 \times 10^{-3}} = 7250 \text{ N/m}^2 = \frac{7250 \text{ N}}{10^4 \text{ cm}^2} = 0.725 \text{ N/cm}^2$$

$\therefore$ Pressure inside the droplet $= p +$ Pressure outside the droplet

$= 0.725 + 10.32 =$ **11.045 N/cm$^2$. Ans.**

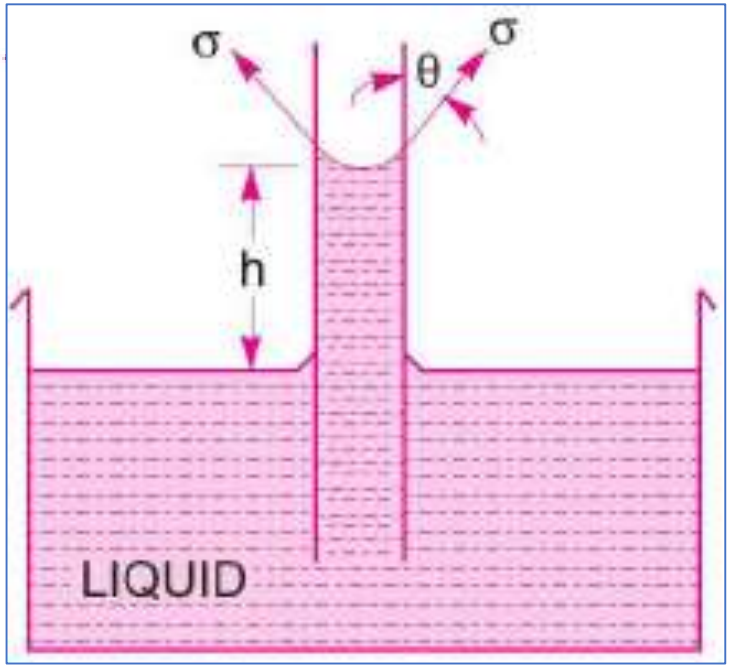
- ✓ **Capillary action** (sometimes capillarity, capillary motion, capillary effect, or wicking) is the ability of a liquid to flow in narrow spaces without the assistance of, or even in opposition to, external forces like gravity.
- ✓ The effect can be seen in the drawing up of liquids between the hairs of a paint-brush, in a thin tube, in porous materials such as paper and plaster, in some non-porous materials such as sand and liquefied carbon fiber, or in a biological cell. It occurs because of intermolecular forces between the liquid and surrounding solid surfaces.
- ✓ If the diameter of the tube is sufficiently small, then the combination of ST (which is caused by cohesion within the liquid) and adhesive forces between the liquid and container wall act to propel the liquid.

- ✓ Capillary action is the movement of a liquid through or along another material against an opposing force, such as gravity.
- ✓ Equilibrium of ST force and gravitational pull on the water cylinder of heighth produces:
- ✓ **Examples** of capillary action in water include water moving up a *straw or glass tube, moving through a paper or cloth towel, moving through a plant, and tears moving through tear ducts*.




- ❑ Capillary penetration in porous media shares its dynamic mechanism with flow in hollow tubes, as both processes are resisted by viscous forces. Consequently, a common apparatus used to demonstrate the phenomenon is the capillary tube. When the lower end of a glass tube is placed in a liquid, such as water, a concave meniscus forms. Adhesion occurs between the fluid and the solid inner wall pulling the liquid column along until there is a sufficient mass of liquid for gravitational forces to overcome these intermolecular forces.
- ❑ The contact length (around the edge) between the top of the liquid column and the tube is proportional to the radius of the tube, while the weight of the liquid column is proportional to the square of the tube's radius. So, a narrow tube will draw a liquid column along further than a wider tube will, given that the inner water molecules cohere sufficiently to the outer ones.

**Expression for Capillary Rise.** Consider a glass tube of small diameter '*d*' opened at both ends and is inserted in a liquid, say water. The liquid will rise in the tube above the level of the liquid.




Let $h$ = height of the liquid in the tube. Under a state of equilibrium, the weight of liquid of height $h$ is balanced by the force at the surface of the liquid in the tube. But the force at the surface of the liquid in the tube is due to surface tension.

Let $\sigma$ = Surface tension of liquid

$\theta$ = Angle of contact between liquid and glass tube.

The weight of liquid of height $h$ in the tube = (Area of tube $\times$ $h$) $\times$ $\rho$ $\times$ $g$

$$= \frac{\pi}{4} d^2 \times h \times \rho \times g \qquad ...(1.17)$$

where $\rho$ = Density of liquid

Vertical component of the surface tensile force

$$= (\sigma \times \text{Circumference}) \times \cos\theta$$

$$= \sigma \times \pi d \times \cos\theta \qquad ...(1.18)$$

For equilibrium, equating (1.17) and (1.18), we get

$$\frac{\pi}{4} d^2 \times h \times \rho \times g = \sigma \times \pi d \times \cos\theta$$

or
$$h = \frac{\sigma \times \pi d \times \cos\theta}{\frac{\pi}{4} d^2 \times \rho \times g} = \frac{4\sigma \cos\theta}{\rho \times g \times d} \quad \ldots(1.19)$$

The value of θ between water and clean glass tube is approximately equal to zero and hence cos θ is equal to unity. Then rise of water is given by

$$h = \frac{4\sigma}{\rho \times g \times d} \quad \ldots(1.20)$$

**Expression for Capillary Fall**. If the glass tube is dipped in mercury, the level of mercury in the tube will be lower than the general level of the outside liquid as shown in Fig. 1.14.

Let $h$ = Height of depression in tube.

Then in equilibrium, two forces are acting on the mercury inside the tube. First one is due to surface tension acting in the downward direction and is equal to $\sigma \times \pi d \times \cos\theta$.

Second force is due to hydrostatic force acting upward and is equal to intensity of pressure at a depth '$h$' × Area

$$= p \times \frac{\pi}{4} d^2 = \rho g \times h \times \frac{\pi}{4} d^2 \{\because p = \rho g h\}$$

Equating the two, we get

$$\sigma \times \pi d \times \cos\theta = \rho g h \times \frac{\pi}{4} d^2$$

$$\therefore \quad h = \frac{4\sigma \cos\theta}{\rho g d} \quad \ldots(1.21)$$

Value of θ for mercury and glass tube is 128°.

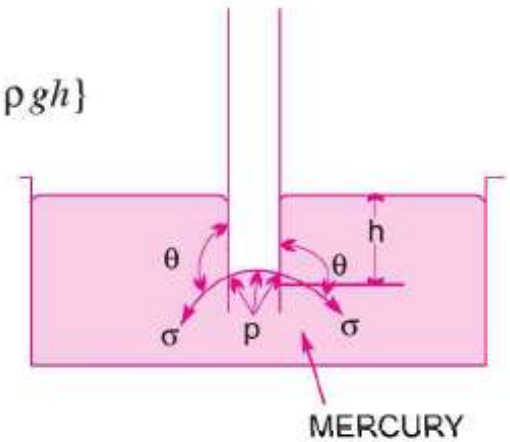



Fig. 1.14

**Problem 13:** Calculate the capillary rise in a glass tube of 2.5 mm diameter when immersed vertically in water and mercury. Take surface tensions $\sigma$ = 0.0725 N/m for water, and $\sigma$ = 0.52 N/m for mercury in contact with air. Specific gravity for mercury is given as 13.6 and angle of contact with glass is = 130°.

Dia. of tube, $d = 2.5 \text{ mm} = 2.5 \times 10^{-3} \text{ m}$

Surface tension, $\sigma$ for water $= 0.0725$ N/m

$\sigma$ for mercury $= 0.52$ N/m

Sp. gr. of mercury $= 13.6$

$\therefore$ Density $= 13.6 \times 1000 \text{ kg/m}^3$.

**(*a*) Capillary rise for water ($\theta = 0^o$)**

Using equation (1.20), we get $h = \dfrac{4\sigma}{\rho \times g \times d} = \dfrac{4 \times 0.0725}{1000 \times 9.81 \times 2.5 \times 10^{-3}}$

$= .0118$ m $=$ **1.18 cm. Ans.**

**(*b*) For mercury**

Angle of contact between mercury and glass tube, $\theta = 130°$

Using equation (1.21), we get $h = \dfrac{4\sigma \cos\theta}{\rho \times g \times d} = \dfrac{4 \times 0.52 \times \cos 130°}{13.6 \times 1000 \times 9.81 \times 2.5 \times 10^{-3}}$

$= -.004$ m $=$ **– 0.4 cm. Ans.**

The negative sign indicates the capillary depression.

**Problem 14:** *Calculate the capillary effect in millimetres in a glass tube of 4 mm diameter, when immersed in (i) water, and (ii) mercury. The temperature of the liquid is 20°C and the values of the surface tension of water and mercury at 20°C in contact with air are 0.073575 N/m and 0.51 N/m respectively. The angle of contact for water is zero and that for mercury is 130°. Take density of water at 20°C as equal to 998 kg/m$^3$.*

**Solution.** Given :

Dia. of tube, $d = 4 \text{ mm} = 4 \times 10^{-3} \text{ m}$

The capillary effect (*i.e.*, capillary rise or depression) is given by equation (1.20) as

$$h = \frac{4\sigma \cos\theta}{\rho \times g \times d}$$

where $\sigma$ = surface tension in N/m

$\theta$ = angle of contact, and $\rho$ = density

(*i*) **Capillary effect for water**

$\sigma = 0.073575$ N/m, $\theta = 0°$

$\rho = 998$ kg/m$^3$ at 20°C

$$\therefore \quad h = \frac{4 \times 0.073575 \times \cos 0°}{998 \times 9.81 \times 4 \times 10^{-3}} = 7.51 \times 10^{-3} \text{ m} = \textbf{7.51 mm. Ans.}$$

(*ii*) **Capillary effect for mercury**

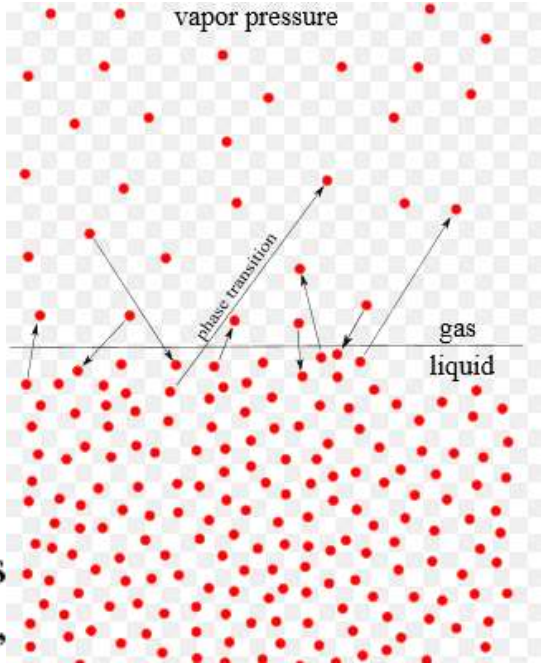
$\sigma = 0.51$ N/m, $\theta = 130°$ and

$\rho = \text{sp. gr.} \times 1000 = 13.6 \times 1000 = 13600$ kg/m$^2$

$$\therefore \quad h = \frac{4 \times 0.51 \times \cos 130°}{13600 \times 9.81 \times 4 \times 10^{-3}} = -2.46 \times 10^{-3} \text{ m} = \textbf{– 2.46 mm. Ans.}$$

The negative sign indicates the capillary depression.

# VAPOUR PRESSURE



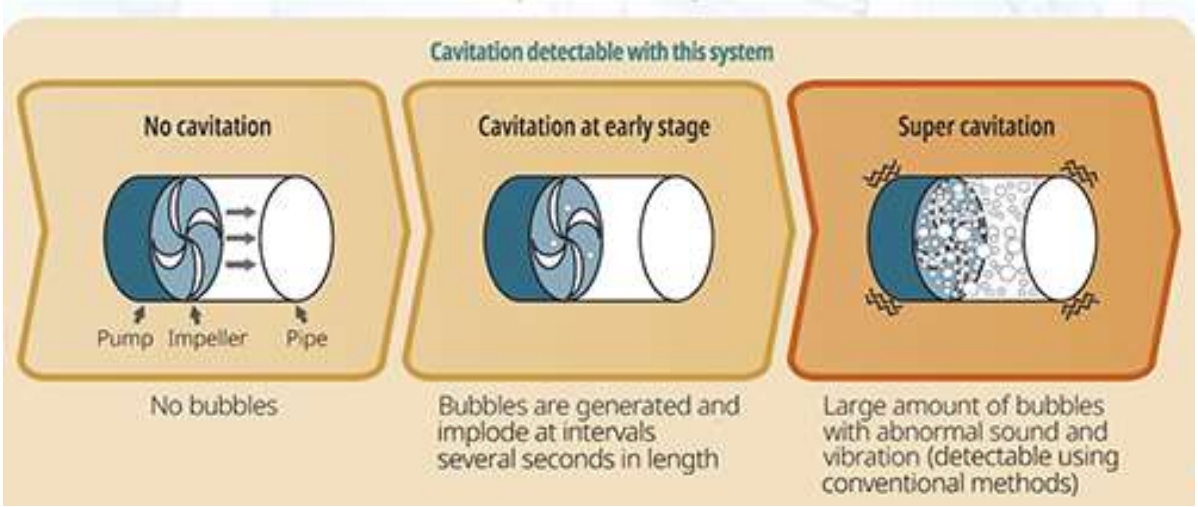

- Vapor pressure or equilibrium vapor pressure is defined as the pressure exerted by a vapor in thermodynamic equilibrium with its condensed phases (solid or liquid) at a given temperature in a closed system.
- The equilibrium vapor pressure is an indication of a liquid's evaporation rate. It relates to the tendency of particles to escape from the liquid (or a solid). A substance with a high vapor pressure at normal temperatures is often referred to as volatile. The pressure exhibited by vapor present above a liquid surface is known as vapor pressure.

Consider a liquid (say water) which is confined in a closed vessel. Let the temperature of liquid is 20°C and pressure is atmospheric. This liquid will vaporise at 100°C. When vaporization takes place, the molecules escapes from the free surface of the liquid. These vapour molecules get accumulated in the space between the free liquid surface and top of the vessel. These accumulated vapours exert a pressure on the liquid surface. This pressure is known as **vapour pressure** of the liquid or this is the pressure at which the liquid is converted into vapours.

Again consider the same liquid at 20°C at atmospheric pressure in the closed vessel. If the pressure above the liquid surface is reduced by some means, the boiling temperature will also reduce. If the pressure is reduced to such an extent that it becomes equal to or less than the vapour pressure, the boiling of the liquid will start, though the temperature of the liquid is 20°C. Thus a liquid may boil even at ordinary temperature, if the pressure above the liquid surface is reduced so as to be equal or less than the vapour pressure of the liquid at that temperature.

# Cavitation

- Cavitation is a phenomenon in which rapid changes of pressure in a liquid lead to the formation of small vapor-filled cavities in places where the pressure is relatively low.
- Cavitation occurs when the liquid in a pump turns to a vapor at low pressure. It occurs because there is not enough pressure at the suction end of the pump, or insufficient Net Positive Suction Head available (NPSHa). When cavitation takes place, air bubbles are created at low pressure.




- The cavitation cause erosion of surface as well as noise and vibrations.



Now consider a flowing liquid in a system. If the pressure at any point in this flowing liquid becomes equal to or less than the vapour pressure, the vaporization of the liquid starts. The bubbles of these vapours are carried by the flowing liquid into the region of high pressure where they collapse, giving rise to high impact pressure. The pressure developed by the collapsing bubbles is so high that the material from the adjoining boundaries gets eroded and cavities are formed on them. This phenomenon is known as **cavitation**.

# PASCAL'S LAW

It states that the pressure or intensity of pressure at a point in a static fluid is equal in all directions. This is proved as :

The fluid element is of very small dimensions *i.e.*, *dx*, *dy* and *ds*.

Consider an arbitrary fluid element of wedge shape in a fluid mass at rest as shown in Fig. 2.1. Let the width of the element perpendicular to the plane of paper is unity and $p_x$, $p_y$ and $p_z$ are the pressures or intensity of pressure acting on the face *AB*, *AC* and *BC* respectively. Let $\angle ABC = \theta$. Then the forces acting on the element are :

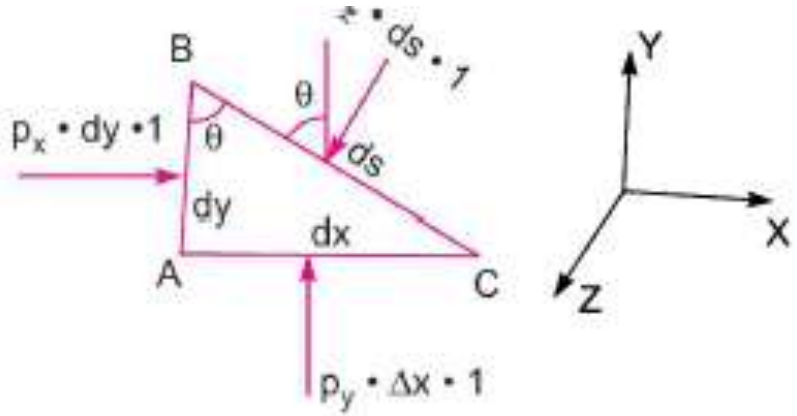



Fig. 2.1 *Forces on a fluid element.*

1. Pressure forces normal to the surfaces, and
2. Weight of element in the vertical direction.

The forces on the faces are :

Force on the face *AB* $= p_x \times$ Area of face *AB*

$= p_x \times dy \times 1$

Similarly force on the face *AC* $= p_y \times dx \times 1$

Force on the face *BC* $= p_z \times ds \times 1$

Weight of element = (Mass of element) $\times g$

$$= (\text{Volume} \times \rho) \times g = \left(\frac{AB \times AC}{2} \times 1\right) \times \rho \times g,$$

where $\rho$ = density of fluid.

Resolving the forces in $x$-direction, we have

$$p_x \times dy \times 1 - p\,(ds \times 1)\sin(90^\circ - \theta) = 0$$

or $$p_x \times dy \times 1 - p_z\, ds \times 1 \cos\theta = 0.$$

But from Fig. 2.1, $$ds\cos\theta = AB = dy$$

$\therefore$ $$p_x \times dy \times 1 - p_z \times dy \times 1 = 0$$

or $$p_x = p_z \qquad ...(2.1)$$

Similarly, resolving the forces in $y$-direction, we get

$$p_y \times dx \times 1 - p_z \times ds \times 1 \cos(90^\circ - \theta) - \frac{dx \times dy}{2} \times 1 \times \rho \times g = 0$$

or $$p_y \times dx - p_z\, ds \sin\theta - \frac{dxdy}{2} \times \rho \times g = 0.$$

But $ds \sin\theta = dx$ and also the element is very small and hence weight is negligible.

$\therefore$ $$p_y dx - p_z \times dx = 0$$

or $$p_y = p_z \qquad ...(2.2)$$

From equations (2.1) and (2.2), we have

$$p_x = p_y = p_z \qquad ...(2.3)$$

The above equation shows that the pressure at any point in $x$, $y$ and $z$ directions is equal.

Since the choice of fluid element was completely arbitrary, which means the pressure at any point is the same in all directions.

## ▶ 2.3 PRESSURE VARIATION IN A FLUID AT REST

The pressure at any point in a fluid at rest is obtained by the Hydrostatic Law which states that the rate of increase of pressure in a vertically downward direction must be equal to the specific weight of the fluid at that point. This is proved as :

Consider a small fluid element as shown in Fig. 2.2

Let $\Delta A$= Cross-sectional area of element

$\Delta Z$ = Height of fluid element

$p$ = Pressure on face $AB$

$Z$ = Distance of fluid element from free surface.

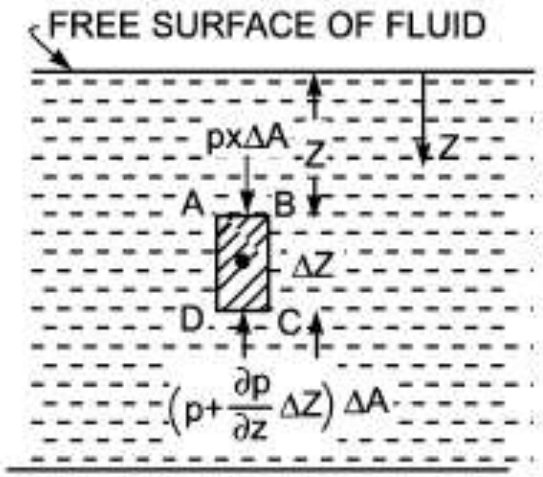



Fig. 2.2 *Forces on a fluid element.*

The forces acting on the fluid element are :

1. Pressure force on $AB = p \times \Delta A$ and acting perpendicular to face $AB$ in the downward direction.
2. Pressure force on $CD = \left(p + \frac{\partial p}{\partial Z}\Delta Z\right) \times \Delta A$, acting perpendicular to face $CD$, vertically upward direction.
3. Weight of fluid element = Density $\times g \times$ Volume = $\rho \times g \times (\Delta A \times \Delta Z)$.
4. Pressure forces on surfaces $BC$ and $AD$ are equal and opposite. For equilibrium of fluid element, we have

$$p\Delta A - \left(p + \frac{\partial p}{\partial Z}\Delta Z\right)\Delta A + \rho \times g \times (\Delta A \times \Delta Z) = 0$$

or $$p\Delta A - p\Delta A - \frac{\partial p}{\partial Z}\Delta Z\Delta A + \rho \times g \times \Delta A \times Z = 0$$

or $$-\frac{\partial p}{\partial Z}\Delta Z\Delta A + \rho \times g \times \Delta A\Delta Z = 0$$

or $$\frac{\partial p}{\partial Z}\Delta Z\Delta A = \rho \times g \times \Delta A\Delta Z \quad \text{or} \quad \frac{\partial p}{\partial Z} = \rho \times g \text{ [cancelling } \Delta A\Delta Z \text{ on both sides]}$$

$$\therefore \quad \frac{\partial p}{\partial Z} = \rho \times g = w \qquad (\because \ \rho \times g = w) \qquad \ldots(2.4)$$

where $w$ = Weight density of fluid.

Equation (2.4) states that rate of increase of pressure in a vertical direction is equal to weight density of the fluid at that point. This is **Hydrostatic Law.**

By integrating the above equation (2.4) for liquids, we get

$$\int dp = \int \rho g dZ$$

or $$p = \rho g Z \qquad \ldots(2.5)$$

where $p$ is the pressure above atmospheric pressure and $Z$ is the height of the point from free surfaces.

From equation (2.5), we have $$Z = \frac{p}{\rho \times g} \qquad \ldots(2.6)$$

Here Z is called **pressure head.**

**Problem 15:** *A hydraulic press has a ram of 30 cm diameter and a plunger of 4.5 cm diameter. Find the weight lifted by the hydraulic press when the force applied at the plunger is 500 N.*

Dia. of ram, $D = 30 \text{ cm} = 0.3 \text{ m}$

Dia. of plunger, $d = 4.5 \text{ cm} = 0.045 \text{ m}$

Force on plunger, $F = 500 \text{ N}$

Find weight lifted $= W$

Area of ram, $$A = \frac{\pi}{4} D^2 = \frac{\pi}{4} (0.3)^2 = 0.07068 \text{ m}^2$$

Area of plunger, $$a = \frac{\pi}{4} d^2 = \frac{\pi}{4} (0.045)^2 = .00159 \text{ m}^2$$

Pressure intensity due to plunger

$$= \frac{\text{Force on plunger}}{\text{Area of plunger}} = \frac{F}{a} = \frac{500}{.00159} \text{ N/m}^2.$$

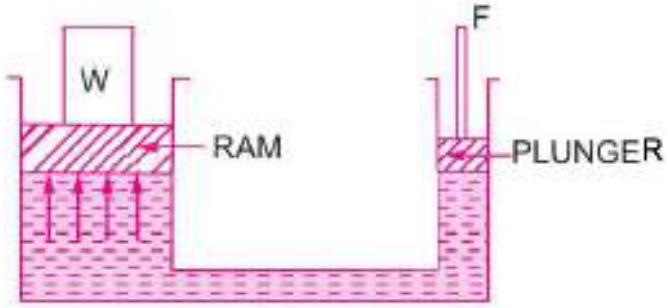


Fig. 2.3

Due to Pascal's law, the intensity of pressure will be equally transmitted in all directions. Hence the pressure intensity at the ram

$$= \frac{500}{.00159} = 314465.4 \text{ N/m}^2$$

But pressure intensity at ram $$= \frac{\text{Weight}}{\text{Area of ram}} = \frac{W}{A} = \frac{W}{.07068} \text{ N/m}^2$$

$$\frac{W}{.07068} = 314465.4$$

$\therefore$ Weight $= 314465.4 \times .07068 = 22222 \text{ N} = \mathbf{22.222 \text{ kN. Ans.}}$

**Problem 16:** *The diameters of a small piston and a large piston of a hydraulic jack are 3 cm and 10 cm respectively. A force of 80 N is applied on the small piston. Find the load lifted by the large piston when :*

*(a) the pistons are at the same level.*

*(b) small piston is 40 cm above the large piston.*

*The density of the liquid in the jack is given as 1000 kg/m$^3$.*

Dia. of small piston, $d = 3$ cm

$\therefore$ Area of small piston, $a = \frac{\pi}{4} d^2 = \frac{\pi}{4} \times (3)^2 = 7.068 \text{ cm}^2$

Dia. of large piston, $D = 10$ cm

$\therefore$ Area of larger piston, $A = \frac{p}{4} \times (10)^2 = 78.54 \text{ cm}^2$

Force on small piston, $F = 80$ N

Let the load lifted $= W$.

**(a) When the pistons are at the same level**

Pressure intensity on small piston

$$\frac{F}{a} = \frac{80}{7.068} \text{ N/cm}^2$$

This is transmitted equally on the large piston.

$\therefore$ Pressure intensity on the large piston

$$= \frac{80}{7.068}$$

$\therefore$ Force on the large piston = Pressure × Area

$$= \frac{80}{7.068} \times 78.54 \text{ N} = \mathbf{888.96 \text{ N. Ans.}}$$

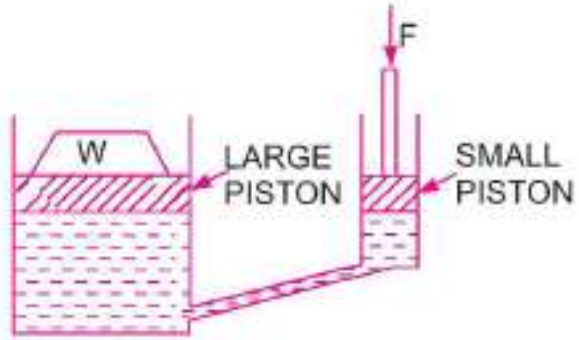



Fig. 2.5

**(*b*) When the small piston is 40 cm above the large piston**

Pressure intensity on the small piston

$$= \frac{F}{a} = \frac{80}{7.068} \frac{\text{N}}{\text{cm}^2}$$

$\therefore$ Pressure intensity at section *A-A*

$$= \frac{F}{a} + \text{Pressure intensity due to height of 40 cm of liquid.}$$

But pressure intensity due to 40 cm of liquid

$$= \rho \times g \times h = 1000 \times 9.81 \times 0.4 \text{ N/m}^2$$

$$= \frac{1000 \times 9.81 \times .40}{10^4} \text{ N/cm}^2 = 0.3924 \text{ N/cm}^2$$

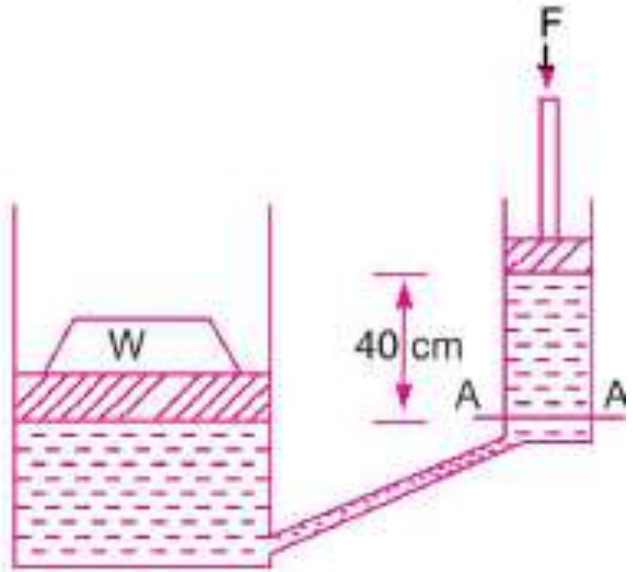



Fig. 2.6

$\therefore$ Pressure intensity at section *A-A*

$$= \frac{80}{7.068} + 0.3924$$

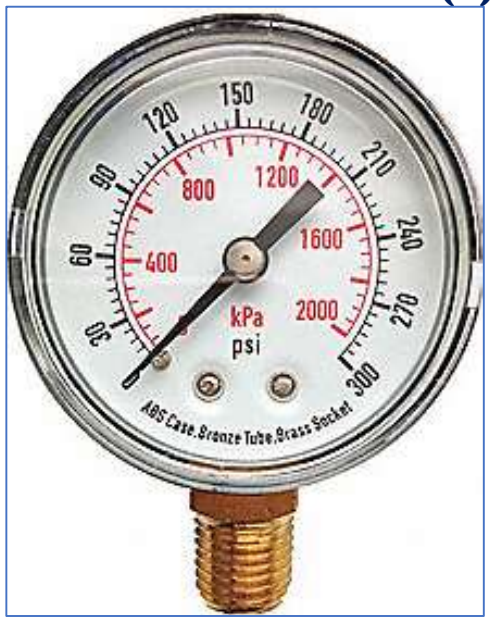
$$= 11.32 + 0.3924 = 11.71 \text{ N/cm}^2$$

$\therefore$ Pressure intensity transmitted to the large piston = 11.71 N/cm$^2$

$\therefore$ Force on the large piston = Pressure $\times$ Area of the large piston

$$= 11.71 \times A = 11.71 \times 78.54 = 919.7 \text{ N.}$$

# PRESSURE & its MEASUREMENT




- ✓ The ***pressure*** is the ratio of ***force*** to the ***area*** over which that force is distributed, it is the force per unit area applied in a direction perpendicular to the surface of an object.

**Why do we measure Pressure:**

- ➢ The measurement and control of fluid pressure is important in process industries a pressure sensor measures pressure, typically of gases or liquids **or** Pressure measurement is the analysis of an applied force by a fluid (liquid or gas) on a surface.
- ➢ Pressure sensors can be used to indirectly measure other variables such as fluid/gas flow, speed, water level, and altitude.




**Types of Pressure:**

**Atmospheric pressure:** It is the pressure that an area experiences due to the force exerted by the atmosphere.

**Gauge pressure:** Gauge pressure is the pressure relative to the atmospheric pressure, it can be described as, how much above or below is the pressure with respect to the atmospheric pressure

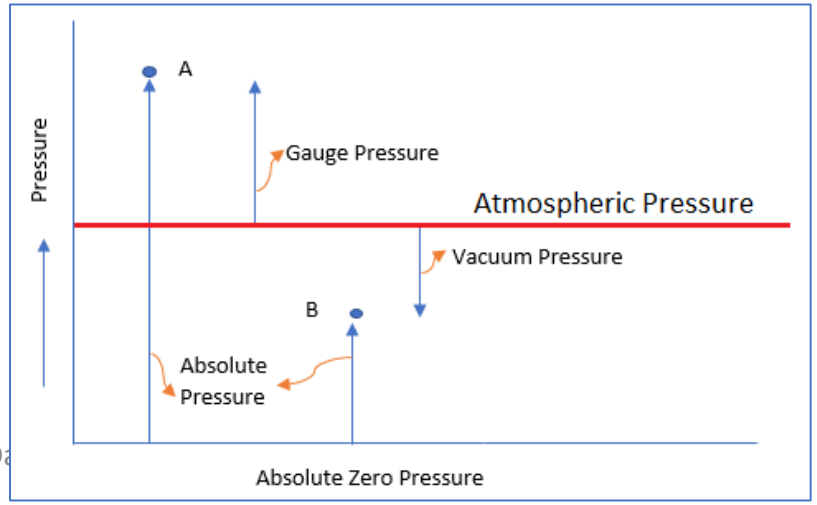
**Vacuum pressure:** Vacuum pressure is the pressure below atmospheric pressure it is measured by using vacuum gauges which will indicate the difference between the atmospheric pressure and the absolute pressure.

**Absolute pressure:** Measured above total vacuum or zero absolute. Zero absolute represents total lack of pressure

| Type | Definition | Application Examples |
|---|---|---|
| Gauge | Reference to atmospheric pressure. | Car Tyre<br>Water Level Measurement<br>Chamber Pressure<br>Hydraulic Applications |
| Sealed | Referenced to a sealed chamber closed with atmospheric pressure. | For use in Aggressive Media<br>Industrial Applications<br>Washdown Environments<br>Food and Beverage |
| Absolute | The reference is a vacuum (0 bar or no pressure). | Barometric<br>Weather Stations and Meteorological Applications |
| Differential | Measuring the difference between two pressure port readings. | Filter and Pump Monitoring<br>Air Conditioner<br>HVAC Heating Ventilation and Air Conditioning<br>Clean Room Monitoring |

**Differential pressure:** It can be defined as the difference in magnitude between some pressure value and some reference pressure. Absolute pressure can be considered as a differential pressure with a total vacuum or zero absolute as the reference, gauge pressure could be considered as differential pressure with atmospheric pressure as reference.

**Static and dynamic pressure:** Static pressure is uniform in all directions, so pressure measurements are independent on the direction in an immovable fluid flow. If pressure is applied on the surface perpendicular to the flow direction while having a little impact on the surface parallel to flow direction and this directional component which is present in a moving fluid can be called as **dynamic pressure.**



**Problem 17:** *What are the gauge pressure and absolute pressure at a point 3 m below the free surface of a liquid having a density of $1.53 \times 10^3$ kg/m$^3$ if the atmospheric pressure is equivalent to 750 mm of mercury ? The specific gravity of mercury is 13.6 and density of water = 1000 kg/m$^3$.*

**Solution.** Given :

Depth of liquid, $Z_1 = 3$ m

Density of liquid, $\rho_1 = 1.53 \times 10^3$ kg/m$^3$

Atmospheric pressure head, $Z_0 = 750$ mm of Hg

$$= \frac{750}{1000} = 0.75 \text{ m of Hg}$$

$\therefore$ Atmospheric pressure, $p_{atm} = \rho_0 \times g \times Z_0$

where $\rho_0$ = Density of Hg = Sp. gr. of mercury × Density of water = $13.6 \times 1000$ kg/m$^3$

and $Z_0$ = Pressure head in terms of mercury.

$\therefore$ $p_{atm} = (13.6 \times 1000) \times 9.81 \times 0.75$ N/m$^2$ ($\because$ $Z_0 = 0.75$)

$= 100062$ N/m$^2$

Pressure at a point, which is at a depth of 3 m from the free surface of the liquid is given by,

$p = \rho_1 \times g \times Z_1$

$= (1.53 \times 1000) \times 9.81 \times 3 = 45028$ N/m$^2$

$\therefore$ Gauge pressure, $p =$ **45028 N/m$^2$. Ans.**

Now absolute pressure = Gauge pressure + Atmospheric pressure

= 45028 + 100062 = **145090 N/m$^2$. Ans.**

# PRESSURE MEASUREMENT

The pressure of a fluid is measured by the following devices

1. Manometers
2. Mechanical Gauges.

**2.5.1 Manometers.** Manometers are defined as the devices used for measuring the pressure at a point in a fluid by balancing the column of fluid by the same or another column of the fluid. They are classified as :

(*a*) Simple Manometers, (*b*) Differential Manometers.

**2.5.2 Mechanical Gauges.** Mechanical gauges are defined as the devices used for measuring the pressure by balancing the fluid column by the spring or dead weight. The commonly used mechanical pressure gauges are :

(*a*) Diaphragm pressure gauge, (*b*) Bourdon tube pressure gauge,
(*c*) Dead-weight pressure gauge, and (*d*) Bellows pressure gauge.

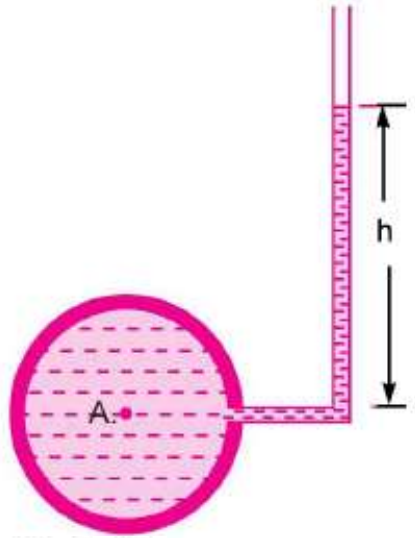
## SIMPLE MANOMETER

A simple manometer consists of a glass tube having one of its ends connected to a point where pressure is to be measured and other end remains open to atmosphere. Common types of simple manometers are :

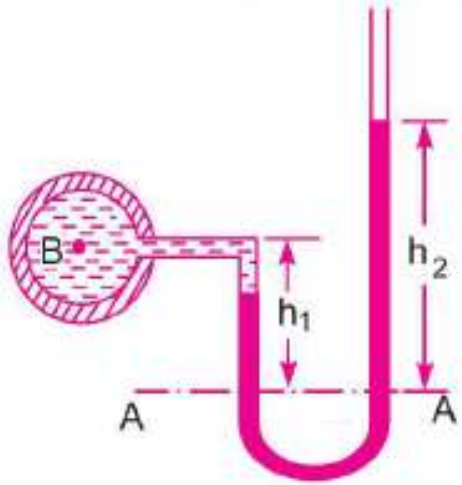
1. Piezometer,
2. U-tube Manometer, and
3. Single Column Manometer.

**2.6.1 Piezometer.** It is the simplest form of manometer used for measuring gauge pressures. One end of this manometer is connected to the point where pressure is to be measured and other end is open to the atmosphere as shown in Fig. 2.8. The rise of liquid gives the pressure head at that point. If at a point $A$, the height of liquid say water is $h$ in piezometer tube, then pressure at $A$

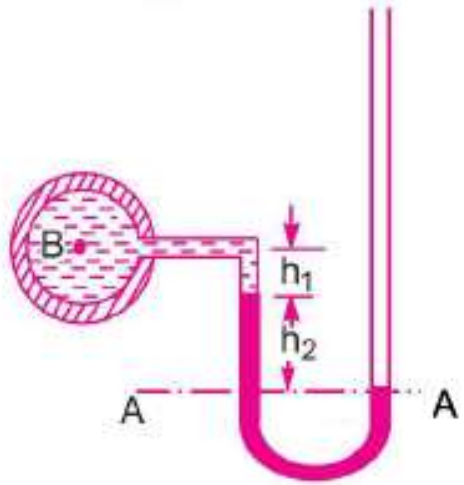
$$= \rho \times g \times h \ \frac{\text{N}}{\text{m}^2}.$$




**2.6.2 U-tube Manometer.** It consists of glass tube bent in U-shape, one end of which is connected to a point at which pressure is to be measured and other end remains open to the atmosphere as shown in Fig. 2.9. The tube generally contains mercury or any other liquid whose specific gravity is greater than the specific gravity of the liquid whose pressure is to be measured.




(a) For gauge pressure




(b) For vacuum pressure

(*a*) **For Gauge Pressure.** Let $B$ is the point at which pressure is to be measured, whose value is $p$. The datum line is $A$-$A$.

Let $h_1$ = Height of light liquid above the datum line
$h_2$ = Height of heavy liquid above the datum line
$S_1$ = Sp. gr. of light liquid
$\rho_1$ = Density of light liquid = $1000 \times S_1$
$S_2$ = Sp. gr. of heavy liquid
$\rho_2$ = Density of heavy liquid = $1000 \times S_2$

As the pressure is the same for the horizontal surface. Hence pressure above the horizontal datum line $A$-$A$ in the left column and in the right column of U-tube manometer should be same.

Pressure above $A$-$A$ in the left column $= p + \rho_1 \times g \times h_1$

Pressure above $A$-$A$ in the right column $= \rho_2 \times g \times h_2$

Hence equating the two pressures $p + \rho_1 g h_1 = \rho_2 g h_2$

$\therefore$ $p = (\rho_2 g h_2 - \rho_1 \times g \times h_1)$. ...(2.7)

(*b*) **For Vacuum Pressure.** For measuring vacuum pressure, the level of the heavy liquid in the manometer will be as shown in Fig. 2.9 (*b*). Then

Pressure above $A$-$A$ in the left column $= \rho_2 g h_2 + \rho_1 g h_1 + p$

Pressure head in the right column above $A$-$A$ $= 0$

$\therefore$ $\rho_2 g h_2 + \rho_1 g h_1 + p = 0$

$\therefore$ $p = -(\rho_2 g h_2 + \rho_1 g h_1)$. ...(2.8)

**Problem 18:** *The right limb of a simple U-tube manometer containing mercury is open to the atmosphere while the left limb is connected to a pipe in which a fluid of sp. gr. 0.9 is flowing. The centre of the pipe is 12 cm below the level of mercury in the right limb. Find the pressure of fluid in the pipe if the difference of mercury level in the two limbs is 20 cm.*

**Solution.** Given :

Sp. gr. of fluid, $S_1 = 0.9$

$\therefore$ Density of fluid, $\rho_1 = S_1 \times 1000 = 0.9 \times 1000 = 900 \text{ kg/m}^3$

Sp. gr. of mercury, $S_2 = 13.6$

$\therefore$ Density of mercury, $\rho_2 = 13.6 \times 1000 \text{ kg/m}^3$

Difference of mercury level, $h_2 = 20 \text{ cm} = 0.2 \text{ m}$

Height of fluid from $A$-$A$, $h_1 = 20 - 12 = 8 \text{ cm} = 0.08 \text{ m}$

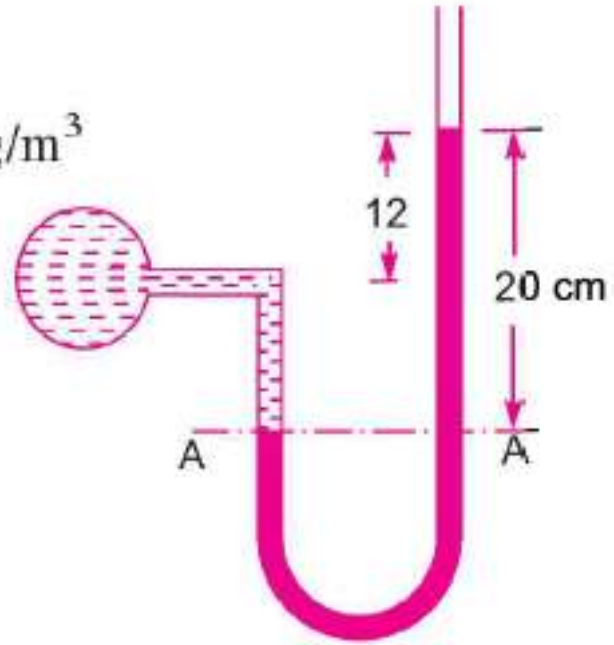



Fig. 2.10

Let $p$ = Pressure of fluid in pipe

Equating the pressure above $A$-$A$, we get

$$p + \rho_1 g h_1 = \rho_2 g h_2$$

or $$p + 900 \times 9.81 \times 0.08 = 13.6 \times 1000 \times 9.81 \times .2$$

$$p = 13.6 \times 1000 \times 9.81 \times .2 - 900 \times 9.81 \times 0.08$$

$$= 26683 - 706 = 25977 \text{ N/m}^2 = \mathbf{2.597 \text{ N/cm}^2}. \textbf{ Ans.}$$

**Problem 19:** *A simple U-tube manometer containing mercury is connected to a pipe in which a fluid of sp. gr. 0.8 and having vacuum pressure is flowing. The other end of the manometer is open to atmosphere. Find the vacuum pressure in pipe, if the difference of mercury level in the two limbs is 40 cm and the height of fluid in the left from the centre of pipe is 15 cm below.*

**Solution.** Given :

Sp. gr. of fluid, $S_1 = 0.8$

Sp. gr. of mercury, $S_2 = 13.6$

Density of fluid, $\rho_1 = 800$

Density of mercury, $\rho_2 = 13.6 \times 1000$

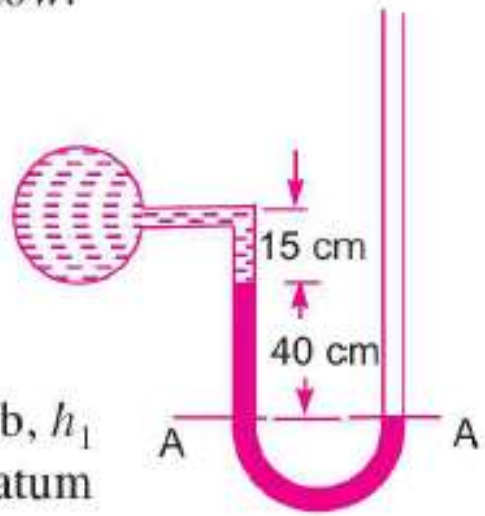


Fig. 2.11

Difference of mercury level, $h_2 = 40$ cm $= 0.4$ m. Height of liquid in left limb, $h_1 = 15$ cm $= 0.15$ m. Let the pressure in pipe $= p$. Equating pressure above datum line $A$-$A$, we get

$$\rho_2 g h_2 + \rho_1 g h_1 + p = 0$$

$$p = -[\rho_2 g h_2 + \rho_1 g h_1]$$

$$= -[13.6 \times 1000 \times 9.81 \times 0.4 + 800 \times 9.81 \times 0.15]$$

$$= -[53366.4 + 1177.2] = -54543.6 \text{ N/m}^2 = \mathbf{-5.454 \text{ N/cm}^2. \text{ Ans.}}$$

**2.6.3 Single Column Manometer.** Single column manometer is a modified form of a U-tube manometer in which a reservoir, having a large cross-sectional area (about 100 times) as compared to the area of the tube is connected to one of the limbs (say left limb) of the manometer as shown in Fig. 2.15. Due to large cross-sectional area of the reservoir, for any variation in pressure, the change in the liquid level in the reservoir will be very small which may be neglected and hence the pressure is given by the height of liquid in the other limb. The other limb may be vertical or inclined. Thus there are two types of single column manometer as :

1. Vertical Single Column Manometer.
2. Inclined Single Column Manometer.

## 1. Vertical Single Column Manometer

Fig. 2.15 shows the vertical single column manometer. Let $X$-$X$ be the datum line in the reservoir and in the right limb of the manometer, when it is not connected to the pipe. When the manometer is connected to the pipe, due to high pressure at $A$, the heavy liquid in the reservoir will be pushed downward and will rise in the right limb.

Let $\Delta h$ = Fall of heavy liquid in reservoir

$h_2$ = Rise of heavy liquid in right limb

$h_1$ = Height of centre of pipe above $X$-$X$

$p_A$ = Pressure at $A$, which is to be measured

$A$ = Cross-sectional area of the reservoir

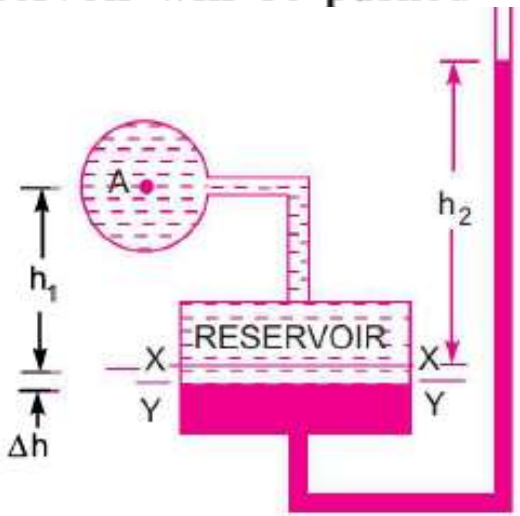
$a$ = Cross-sectional area of the right limb

$S_1$ = Sp. gr. of liquid in pipe

$S_2$ = Sp. gr. of heavy liquid in reservoir and right limb

$\rho_1$ = Density of liquid in pipe

$\rho_2$ = Density of liquid in reservoir



Fall of heavy liquid in reservoir will cause a rise of heavy liquid level in the right limb.

$$\therefore \quad A \times \Delta h = a \times h_2$$

$$\therefore \quad \Delta h = \frac{a \times h_2}{A} \qquad \ldots(i)$$

Now consider the datum line $Y$-$Y$ as shown in Fig. 2.15. Then pressure in the right limb above $Y$-$Y$.

$$= \rho_2 \times g \times (\Delta h + h_2)$$

Pressure in the left limb above $Y$-$Y$ $= \rho_1 \times g \times (\Delta h + h_1) + p_A$

Equating these pressures, we have

$$\rho_2 \times g \times (\Delta h + h_2) = \rho_1 \times g \times (\Delta h + h_1) + p_A$$

or

$$p_A = \rho_2 g\,(\Delta h + h_2) - \rho_1 g(\Delta h + h_1)$$

$$= \Delta h[\rho_2 g - \rho_1 g] + h_2\rho_2 g - h_1\rho_1 g$$

But from equation $(i)$, $\Delta h = \dfrac{a \times h_2}{A}$

$$\therefore \quad p_A = \frac{a \times h_2}{A}\,[\rho_2 g - \rho_1 g] + h_2\rho_2 g - h_1\rho_1 g \qquad \ldots(2.9)$$

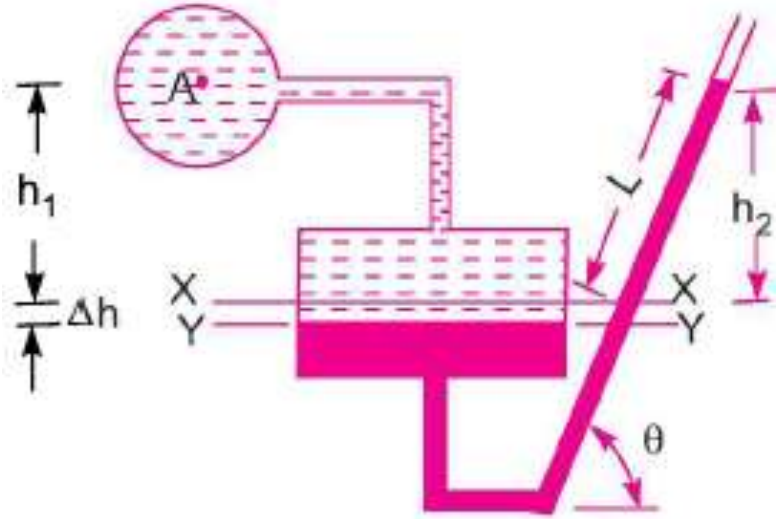
As the area $A$ is very large as compared to $a$, hence ratio $\dfrac{a}{A}$ becomes very small and can be neglected.

Then $p_A = h_2\rho_2 g - h_1\rho_1 g$ $\qquad \ldots(2.10)$

From equation (2.10), it is clear that as $h_1$ is known and hence by knowing $h_2$ or rise of heavy liquid in the right limb, the pressure at $A$ can be calculated.

## 2. Inclined Single Column Manometer

Fig. 2.16 shows the inclined single column manometer. This manometer is more sensitive. Due to inclination the distance moved by the heavy liquid in the right limb will be more.




Let $L$ = Length of heavy liquid moved in right limb from $X$-$X$

$\theta$ = Inclination of right limb with horizontal

$h_2$ = Vertical rise of heavy liquid in right limb from $X$-$X$ = $L \times \sin\theta$

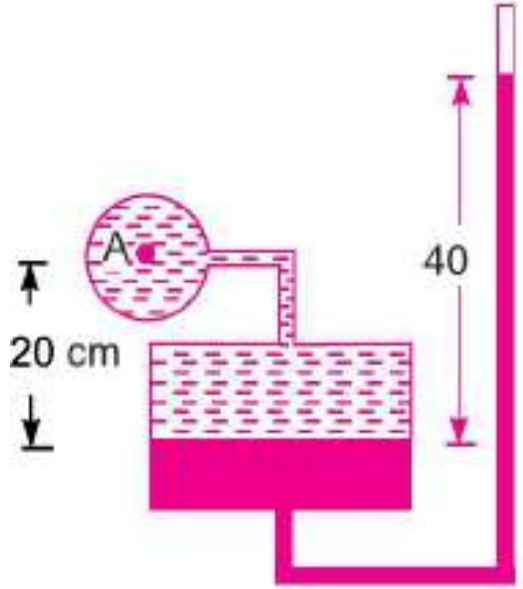
From equation (2.10), the pressure at $A$ is

$$p_A = h_2\rho_2 g - h_1\rho_1 g.$$

Substituting the value of $h_2$, we get

$$p_A = \sin\theta \times \rho_2 g - h_1\rho_1 g.$$

**Problem 20:** *A single column manometer is connected to a pipe containing a liquid of sp. gr. 0.9 as shown in Fig. 2.17. Find the pressure in the pipe if the area of the reservoir is 100 times the area of the tube for the manometer reading shown in Fig. 2.17. The specific gravity of mercury is 13.6.*



Sp. gr. of liquid in pipe, $S_1 = 0.9$

$\therefore$ Density $\rho_1 = 900 \text{ kg/m}^3$

Sp. gr. of heavy liquid, $S_2 = 13.6$

Density, $\rho_2 = 13.6 \times 1000$

$$\frac{\text{Area of reservoir}}{\text{Area of right limb}} = \frac{A}{a} = 100$$

Height of liquid, $h_1 = 20 \text{ cm} = 0.2 \text{ m}$

Rise of mercury in right limb,

$h_2 = 40 \text{ cm} = 0.4 \text{ m}$

Let $p_A$ = Pressure in pipe

Using equation (2.9), we get

$$p_A = \frac{a}{A} h_2[\rho_2 g - \rho_1 g] + h_2\rho_2 g - h_1\rho_1 g$$

$$= \frac{1}{100} \times 0.4[13.6 \times 1000 \times 9.81 - 900 \times 9.81] + 0.4 \times 13.6 \times 1000 \times 9.81 - 0.2 \times 900 \times 9.81$$

$$= \frac{0.4}{100} [133416 - 8829] + 53366.4 - 1765.8$$

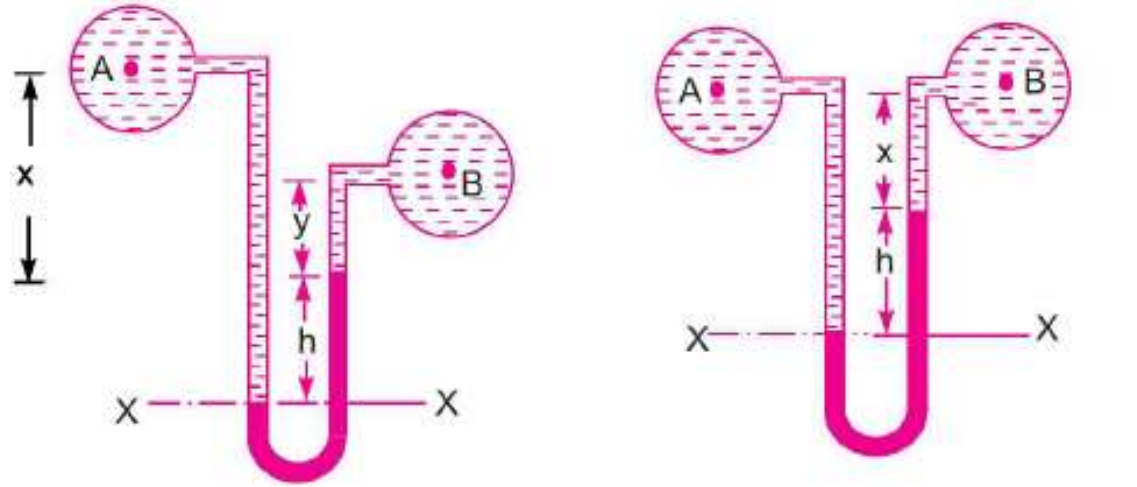
$$= 533.664 + 53366.4 - 1765.8 \text{ N/m}^2 = 52134 \text{ N/m}^2 = \mathbf{5.21 \text{ N/cm}^2}. \textbf{ Ans.}$$

# DIFFERENTIAL MANOMETER

Differential manometers are the devices used for measuring the difference of pressures between two points in a pipe or in two different pipes. A differential manometer consists of a U-tube, containing a heavy liquid, whose two ends are connected to the points, whose difference of pressure is to be measured. Most commonly types of differential manometers are :

1. U-tube differential manometer and
2. Inverted U-tube differential manometer.

**2.7.1 U-tube Differential Manometer.** Fig. 2.18 shows the differential manometers of U-tube type.




(a)Two pipes at different levels (b) A and B are at the same level

Fig. 2.18 *U-tube differential manometers.*

In Fig. 2.18 (*a*), the two points *A* and *B* are at different level and also contains liquids of different sp. gr. These points are connected to the U-tube differential manometer. Let the pressure at *A* and *B* are $p_A$ and $p_B$.

Let $h$ = Difference of mercury level in the U-tube.

$y$ = Distance of the centre of $B$, from the mercury level in the right limb.

$x$ = Distance of the centre of $A$, from the mercury level in the right limb.

$\rho_1$ = Density of liquid at $A$.

$\rho_2$ = Density of liquid at $B$.

$\rho_g$ = Density of heavy liquid or mercury.

Taking datum line at $X$-$X$.

Pressure above $X$-$X$ in the left limb = $\rho_1 g(h + x) + p_A$

where $p_A$ = pressure at $A$.

Pressure above $X$-$X$ in the right limb = $\rho_g \times g \times h + \rho_2 \times g \times y + p_B$

where $p_B$ = Pressure at $B$.

Equating the two pressure, we have

$$\rho_1 g(h + x) + p_A = \rho_g \times g \times h + \rho_2 g y + p_B$$

$$\therefore \quad p_A - p_B = \rho_g \times g \times h + \rho_2 g y - \rho_1 g(h + x)$$

$$= h \times g(\rho_g - \rho_1) + \rho_2 g y - \rho_1 g x \qquad \ldots(2.12)$$

$\therefore$ Difference of pressure at $A$ and $B$ = $h \times g(\rho_g - \rho_1) + \rho_2 g y - \rho_1 g x$

In Fig. 2.18 ($b$), the two points $A$ and $B$ are at the same level and contains the same liquid of density $\rho_1$. Then

Pressure above $X$-$X$ in right limb $= \rho_g \times g \times h + \rho_1 \times g \times x + p_B$

Pressure above $X$-$X$ in left limb $= \rho_1 \times g \times (h + x) + p_A$

Equating the two pressure

$$\rho_g \times g \times h + \rho_1 g x + p_B = \rho_1 \times g \times (h + x) + p_A$$

$$\therefore \quad p_A - p_B = \rho_g \times g \times h + \rho_1 g x - \rho_1 g(h + x)$$

$$= g \times h(\rho_g - \rho_1). \qquad \ldots(2.13)$$

**Problem 21:** *A differential manometer is connected at the two points A and B of two pipes as shown in Fig. 2.19. The pipe A contains a liquid of sp. gr. = 1.5 while pipe B contains a liquid of sp. gr. = 0.9. The pressures at A and B are 1 kgf/cm$^2$ and 1.80 kgf/cm$^2$ respectively. Find the difference in mercury level in the differential manometer.*

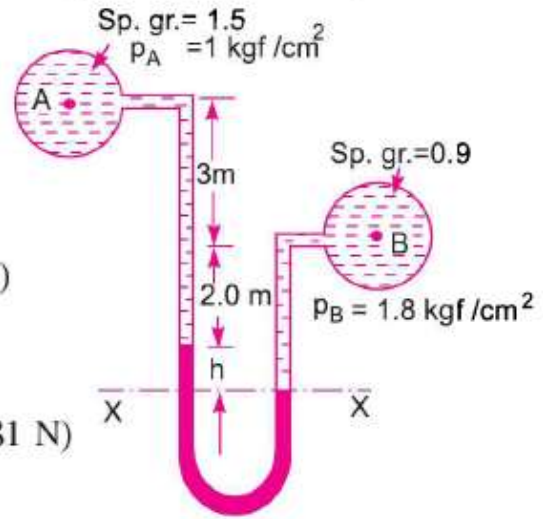



Fig. 2.19

**Solution.** Given :

Sp. gr. of liquid at $A$, $S_1 = 1.5 \quad \therefore \quad \rho_1 = 1500$

Sp. gr. of liquid at $B$, $S_2 = 0.9 \quad \therefore \quad \rho_2 = 900$

Pressure at $A$, $p_A = 1$ kgf/cm$^2$ $= 1 \times 10^4$ kgf/m$^2$

$= 10^4 \times 9.81$ N/m$^2$ ($\because$ 1 kgf = 9.81 N)

Pressure at $B$, $p_B = 1.8$ kgf/cm$^2$

$= 1.8 \times 10^4$ kgf/m$^2$

$= 1.8 \times 10^4 \times 9.81$ N/m$^2$ ($\because$ 1 kgf = 9.81 N)

Density of mercury $= 13.6 \times 1000$ kg/m$^3$

Taking $X$-$X$ as datum line.

Pressure above $X$-$X$ in the left limb

$$= 13.6 \times 1000 \times 9.81 \times h + 1500 \times 9.81 \times (2 + 3) + p_A$$

$$= 13.6 \times 1000 \times 9.81 \times h + 7500 \times 9.81 + 9.81 \times 10^4$$

Pressure above $X$-$X$ in the right limb $= 900 \times 9.81 \times (h + 2) + p_B$

$$= 900 \times 9.81 \times (h + 2) + 1.8 \times 10^4 \times 9.81$$

Equating the two pressure, we get

$$13.6 \times 1000 \times 9.81h + 7500 \times 9.81 + 9.81 \times 10^4 = 900 \times 9.81 \times (h + 2) + 1.8 \times 10^4 \times 9.81$$

Dividing by $1000 \times 9.81$, we get

$$13.6h + 7.5 + 10 = (h + 2.0) \times .9 + 18$$

$$13.6h + 17.5 = 0.9h + 1.8 + 18 = 0.9h + 19.8$$

$$(13.6 - 0.9)h = 19.8 - 17.5 \text{ or } 12.7h = 2.3$$

$\therefore$ $$h = \frac{2.3}{12.7} = 0.181 \text{ m} = \mathbf{18.1 \text{ cm. Ans.}}$$

**2.7.2 Inverted U-tube Differential Manometer.** It consists of an inverted U-tube, containing a light liquid. The two ends of the tube are connected to the points whose difference of pressure is to be measured. It is used for measuring difference of low pressures. Fig. 2.21 shows an inverted U-tube differential manometer connected to the two points $A$ and $B$. Let the pressure at $A$ is more than the pressure at $B$.

Let $h_1$ = Height of liquid in left limb below the datum line $X$-$X$
$h_2$ = Height of liquid in right limb
$h$ = Difference of light liquid
$\rho_1$ = Density of liquid at $A$
$\rho_2$ = Density of liquid at $B$
$\rho_s$ = Density of light liquid
$p_A$ = Pressure at $A$
$p_B$ = Pressure at $B$.

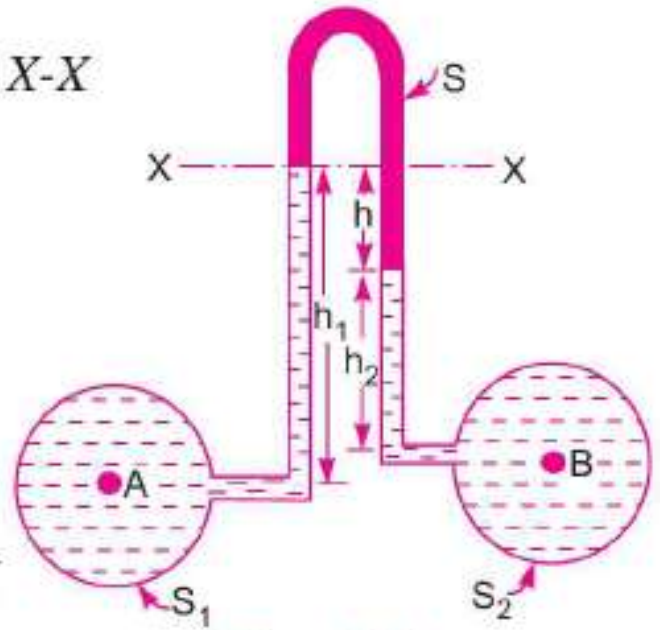


Fig. 2.21

Taking $X$-$X$ as datum line. Then pressure in the left limb below $X$-$X$

$$= p_A - \rho_1 \times g \times h_1.$$

Pressure in the right limb below $X$-$X$

$$= p_B - \rho_2 \times g \times h_2 - \rho_s \times g \times h$$

Equating the two pressure

$$p_A - \rho_1 \times g \times h_1 = p_B - \rho_2 \times g \times h_2 - \rho_s \times g \times h$$

or
$$p_A - p_B = \rho_1 \times g \times h_1 - \rho_2 \times g \times h_2 - \rho_s \times g \times h. \quad ...(2.14)$$

**Problem 22:** *Water is flowing through two different pipes to which an inverted differential manometer having an oil of sp. gr. 0.8 is connected. The pressure head in the pipe A is 2 m of water, find the pressure in the pipe B for the manometer readings as shown in Fig. 2.22.*

**Solution.** Given :

Pressure head at $A = \frac{p_A}{\rho g} = 2$ m of water

$\therefore$ $p_A = \rho \times g \times 2 = 1000 \times 9.81 \times 2 = 19620$ N/m$^2$

Fig. 2.22 shows the arrangement. Taking $X$-$X$ as datum line.

Pressure below $X$-$X$ in the left limb $= p_A - \rho_1 \times g \times h_1$

$= 19620 - 1000 \times 9.81 \times 0.3 = 16677$ N/m$^2$.

Pressure below $X$-$X$ in the right limb

$= p_B - 1000 \times 9.81 \times 0.1 - 800 \times 9.81 \times 0.12$

$= p_B - 981 - 941.76 = p_B - 1922.76$

Equating the two pressure, we get

$16677 = p_B - 1922.76$

$p_B = 16677 + 1922.76 = 18599.76$ N/m$^2$

$p_B = \mathbf{1.8599\ N/cm^2}$. **Ans.**

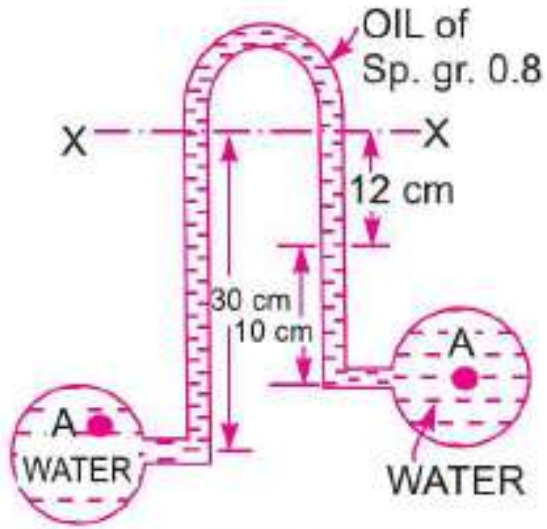



**Fig. 2.22**

**Problem 23:** *In Fig. 2.23, an inverted differential manometer is connected to two pipes A and B which convey water. The fluid in manometer is oil of sp. gr. 0.8. For the manometer readings shown in the figure, find the pressure difference between A and B.*

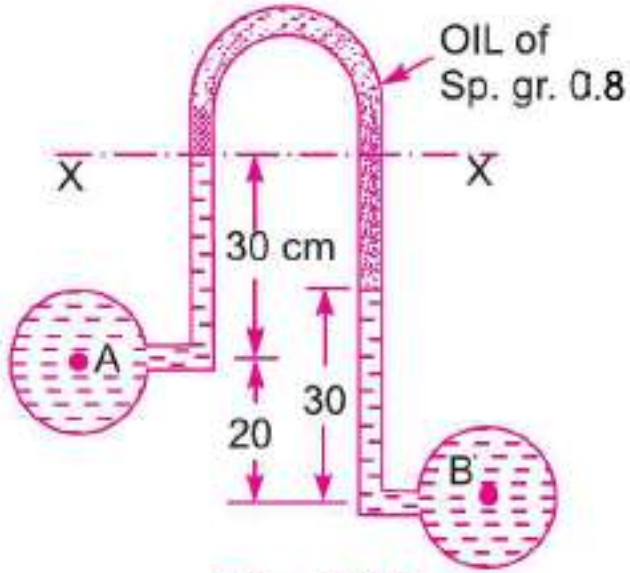


Fig. 2.23

**Solution.** Given :

Sp. gr. of oil $= 0.8 \quad \therefore \quad \rho_s = 800 \text{ kg/m}^3$

Difference of oil in the two limbs

$= (30 + 20) - 30 = 20 \text{ cm}$

Taking datum line at *X-X*

Pressure in the left limb below *X-X*

$= p_A - 1000 \times 9.81 \times 0$

$= p_A - 2943$

Pressure in the right limb below *X-X*

$= p_B - 1000 \times 9.81 \times 0.3 - 800 \times 9.81 \times 0.2$

$= p_B - 2943 - 1569.6 = p_B - 4512.6$

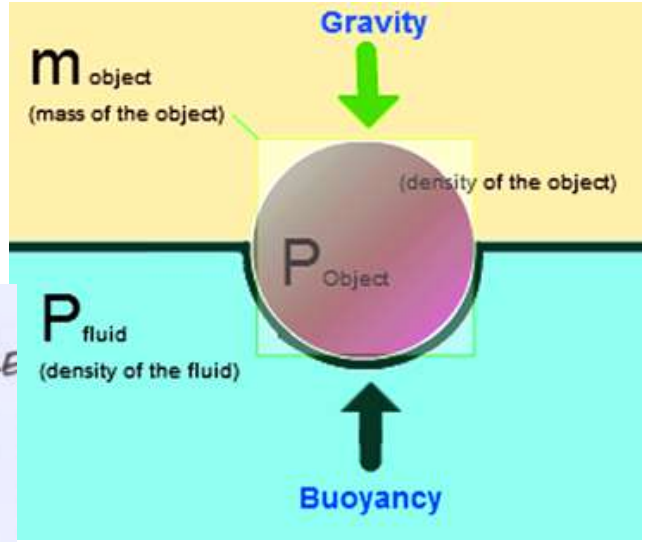
Equating the two pressure $p_A - 2943 = p_B - 4512.6$

$\therefore \quad p_B - p_A = 4512.6 - 2943 = \mathbf{1569.6 \text{ N/m}^2}$. **Ans.**

- ✓ **Buoyancy and Flotation**
- ✓ **Meta center, Stability of floating body, Submerged bodies**
- ✓ **Calculation of metacentric height**
- ✓ **Stability analysis and applications**

# Buoyancy

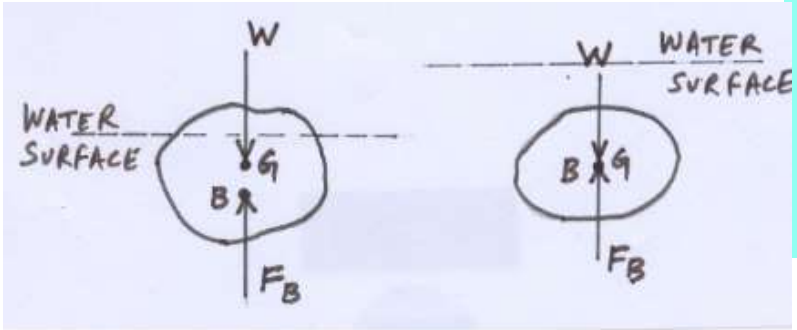
- ✓ **Buoyancy** (also known as the buoyant force) is the force exerted on an object that is wholly or partly immersed in a fluid.
- ✓ **Buoyancy** or **upthrust**, is an upward force exerted by a fluid that opposes the weight of a partially or fully immersed object. In a column of fluid, pressure increases with depth as a result of the weight of the overlying fluid
  - ➢ The symbol for the magnitude of buoyancy is B or $F_B$
  - ➢ As a vector it must be stated with both magnitude and direction.
  - ➢ Buoyancy acts upward for the kind of situations encountered in everyday experience.
  - ➢ As with other forces, the SI unit of buoyancy is the newton [N].




# Centre of Buoyancy

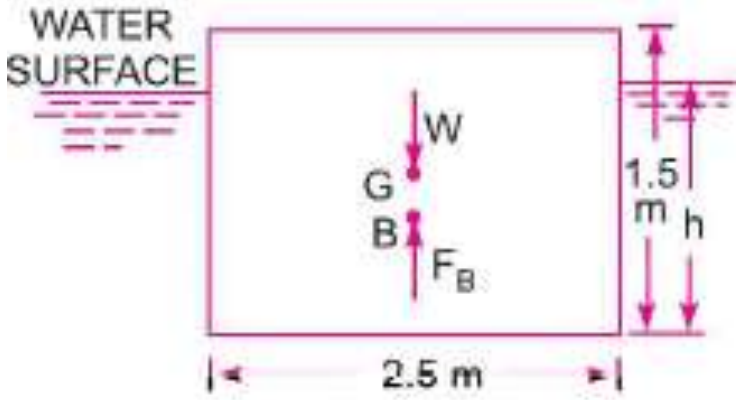
It is defined as the point, through which the force of buoyancy is supposed to act. As the force of buoyancy is a vertical force and is equal to the weight of the fluid displaced by the body, the centre of buoyancy will be the centre of gravity of the fluid displaced.

- ➢ The center of buoyancy is the point where if you were to take all of the displaced fluid and hold it by that point it would remain perfectly balanced, assuming you could hold a fluid in a fixed shape. This point is also called the center of mass. The center of buoyancy for an object is the center of mass for the fluid it displaces




(a) Floating Body (b) Submerged Body

**Problem 24:** Find the volume of the water displaced and the position of Center of Buoyancy for a wooden block of width 2.5 m and depth 1.5 m when it floats horizontally in water. Density of wooden block is 650 kg/m3 and its length is 6.0 m.




Width = 2.5 m
Depth = 1.5 m
Length = 6.0 m
Volume of the block = $2.5 \times 1.5 \times 6.0 = 22.50\ m^3$
Density of wood, $\rho = 650\ kg/m^3$
∴ Weight of block = $\rho \times g \times$ Volume
$= 650 \times 9.81 \times 22.50\ N = 143471\ N$

For equilibrium the weight of water displaced = Weight of wooden block
$= 143471\ N$

∴ Volume of water displaced

$$= \frac{\text{Weight of water displaced}}{\text{Weight density of water}} = \frac{143471}{1000 \times 9.81} = \mathbf{14.625\ m^3.\ Ans.}$$

($\because$ Weight density of water $= 1000 \times 9.81\ N/m^3$)

**Position of Centre of Buoyancy.** Volume of wooden block in water
= Volume of water displaced

r $2.5 \times h \times 6.0 = 14.625\ m^3$, where $h$ is depth of wooden block in water

$$\therefore \quad h = \frac{14.625}{2.5 \times 6.0} = 0.975\ m$$

$$\therefore \quad \text{Centre of Buoyancy} = \frac{0.975}{2} = \mathbf{0.4875\ m\ from\ base.\ Ans.}$$

**Problem 25:** *A wooden log of 0.6 m diameter and 5 m length is floating in river water. Find the depth of the wooden log in water when the sp. gravity of the log is 0.7.*

Dia. of log = 0.6 m

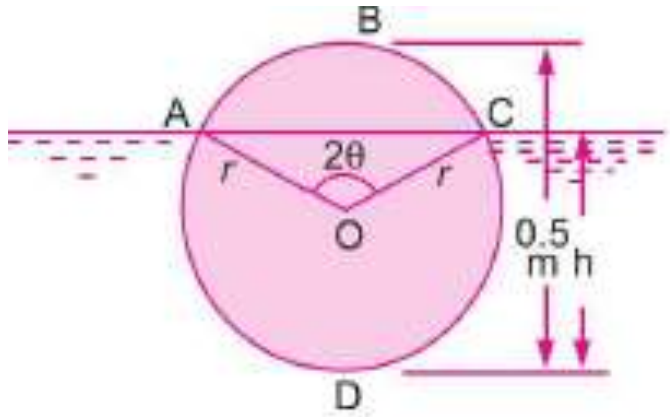
Length, $L = 5$ m

Sp. gr., $S = 0.7$

$\therefore$ Density of log $= 0.7 \times 1000 = 700$ kg/m$^3$

$\therefore$ Weight density of log, $w = \rho \times g$

$= 700 \times 9.81$ N/m$^3$




Find depth of immersion or $h$

Weight of wooden log = Weight density $\times$ Volume of log

$$= 700 \times 9.81 \times \frac{\pi}{4} (D)^2 \times L$$

$$= 700 \times 9.81 \times \frac{\pi}{4} (.6)^2 \times 5 \text{ N} = 989.6 \times 9.81 \text{ N}$$

For equilibrium,

Weight of wooden log = Weight of water displaced

= Weight density of water $\times$ Volume of water displaced

$\therefore$ Volume of water displaced $= \dfrac{989.6 \times 9.81}{1000 \times 9.81} = 0.9896$ m$^3$

($\because$ Weight density of water $= 1000 \times 9.81$ N/m$^3$)

Let $h$ is the depth of immersion

$\therefore$ Volume of log inside water = Area of $ADCA \times$ Length

= Area of $ADCA \times 5.0$

But volume of log inside water = Volume of water displaced = 0.9896 m$^3$

$\therefore$ $0.9896 = \text{Area of } ADCA \times 5.0$

$\therefore$ Area of $ADCA = \dfrac{0.9896}{5.0} = 0.1979 \text{ m}^2$

But area of $ADCA$ = Area of curved surface $ADCOA$ + Area of $\Delta AOC$

$$= \pi r^2 \left[\frac{360^\circ - 2\theta}{360^\circ}\right] + \frac{1}{2} r \cos\theta \times 2r \sin\theta$$

$$= \pi r^2 \left[1 - \frac{\theta}{180^\circ}\right] + r^2 \cos\theta \sin\theta$$

$\therefore$ $$0.1979 = \pi (.3)^2 \left[1 - \frac{\theta}{180^\circ}\right] + (.3)^2 \cos\theta \sin\theta$$

$$0.1979 = .2827 - .00157\,\theta + 0.9 \cos\theta \sin\theta$$

or $$.00157\,\theta - .09 \cos\theta \sin\theta = .2827 - .1979 = 0.0848$$

$$\theta - \frac{.09}{.00157} \cos\theta \sin\theta = \frac{.0848}{.00157}$$

or $$\theta - 57.32 \cos\theta \sin\theta = 54.01.$$

or $$\theta - 57.32 \cos\theta \sin\theta - 54.01 = 0$$

For $\theta = 60^\circ$, $60 - 57.32 \times 0.5 \times .866 - 54.01 = 60 - 24.81 - 54.01 = -18.82$

For $\theta = 70^\circ$, $70 - 57.32 \times .342 \times 0.9396 - 54.01 = 70 - 18.4 - 54.01 = -2.41$

For $\theta = 72^\circ$, $72 - 57.32 \times .309 \times .951 - 54.01 = 72 - 16.84 - 54.01 = +1.14$

For $\theta = 71^\circ$, $71 - 57.32 \times .325 \times .9455 - 54.01 = 71 - 17.61 - 54.01 = -0.376$

$\therefore$ $\theta = 71.5^\circ$, $71.5 - 57.32 \times .3173 \times .948 - 54.01 = 71.5 - 17.24 - 54.01 = +.248$

Then $h = r + r \cos 71.5^\circ$

$= 0.3 + 0.3 \times 0.3173 =$ **0.395 m. Ans.**

# Meta centre:

It is defined as the point about which a body starts oscillating when the body is tilted by a small angle. The meta-centre may also be defined as the point at which the line of action of the force of buoyancy will meet the normal axis of the body when the body is given a small angular displacement.

Consider a body floating in a liquid as shown in Fig. 4.5 (*a*). Let the body is in equilibrium and *G* is the centre of gravity and *B* the centre of buoyancy. For equilibrium, both the points lie on the normal axis, which is vertical.

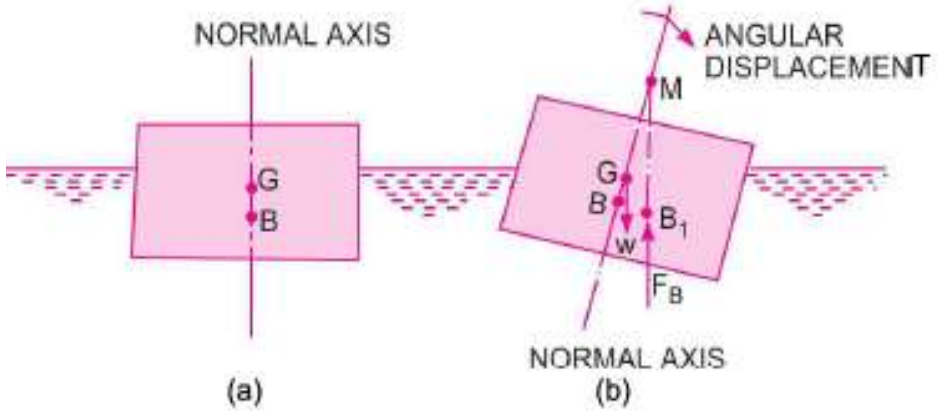



Fig. 4.5 *Meta-centre*

Let the body is given a small angular displacement in the clockwise direction as shown in Fig. 4.5 (*b*). The centre of buoyancy, which is the centre of gravity of the displaced liquid or centre of gravity of the portion of the body sub-merged in liquid, will now be shifted towards right from the normal axis. Let it is at $B_1$ as shown in Fig. 4.5 (*b*). The line of action of the force of buoyancy in this new position, will intersect the normal axis of the body at some point say *M*. This point *M* is called **Meta-centre.**

The distance *MG*, *i.e.*, the distance between the meta-centre of a floating body and the centre of gravity of the body is called meta-centric height.

# ANALYTICAL METHOD FOR META-CENTRE HEIGHT

Fig. 4.6 ($a$) shows the position of a floating body in equilibrium. The location of centre of gravi and centre of buoyancy in this position is at $G$ and $B$. The floating body is given a small angul displacement in the clockwise direction. This is shown in Fig. 4.6 ($b$). The new centre of buoyancy at $B_1$. The vertical line through $B_1$ cuts the normal axis at $M$. Hence $M$ is the me meta-centric height.

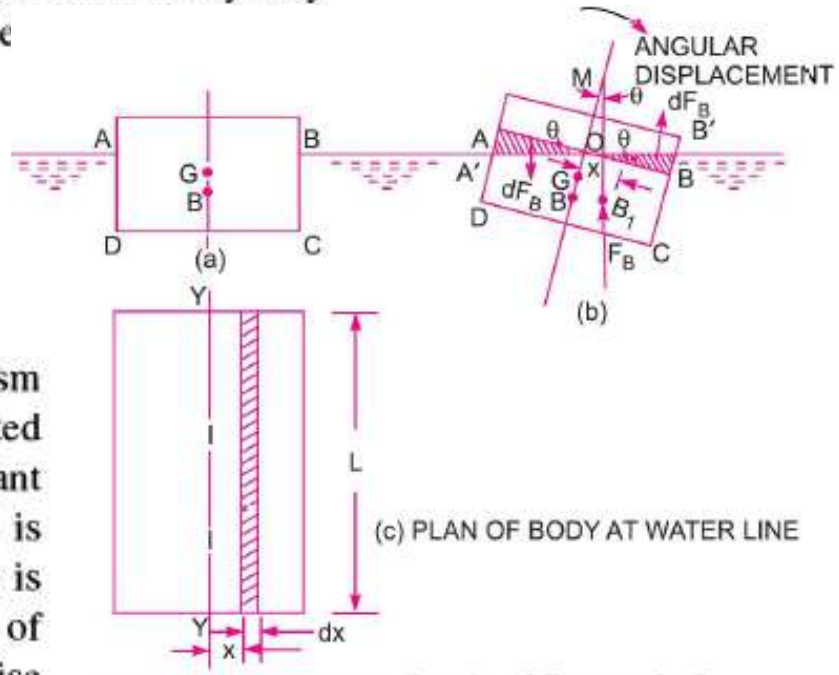



Fig. 4.6 *Meta-centre height of floating body.*

The angular displacement of the body in the clockwise direction causes the wedge-shaped prism $BOB'$ on the right of the axis to go inside the water while the identical wedge-shaped prism represented by $AOA'$ emerges out of the water on the left of the axis. These wedges represent a gain in buoyant force on the right side and a corresponding loss of buoyant force on the left side. The gain is represented by a vertical force $dF_B$ acting through the $C.G.$ of the prism $BOB'$ while the loss is represented by an equal and opposite force $dF_B$ acting vertically downward through the centroid of $AOA'$. The couple due to these buoyant forces $dF_B$ tends to rotate the ship in the counterclockwise direction. Also the moment caused by the displacement of the centre of buoyancy from $B$ to $B_1$ is also in the counterclockwise direction. Thus these two couples must be equal.

**Couple Due to Wedges.** Consider towards the right of the axis a small strip of thickness $dx$ at a distance $x$ from $O$ as shown in Fig. 4.5 ($b$). The height of strip $x \times \angle BOB' = x \times \theta$.

$$\{\because \angle BOB' = \angle AOA' = BMB_1' = \theta\}$$

$\therefore$ Area of strip = Height × Thickness = $x \times \theta \times dx$

If $L$ is the length of the floating body, then

Volume of strip = Area × $L$

$= x \times \theta \times L \times dx$

$\therefore$ Weight of strip = $\rho g \times$ Volume = $\rho g x\, \theta L\, dx$

Similarly, if a small strip of thickness $dx$ at a distance $x$ from $O$ towards the left of the axis is considered, the weight of strip will be $\rho g x \theta\, L\, dx$. The two weights are acting in the opposite direction and hence constitute a couple.

Moment of this couple = Weight of each strip × Distance between these two weights

$= \rho g x\, \theta L\, dx\, [x + x]$

$= \rho g x\, \theta L\, dx \times 2x = 2\rho g x^2\, \theta L\, dx$

$\therefore$ Moment of the couple for the whole wedge

$= \int 2\rho g x^2\, \theta L\, dx$ ...(4.1)

Moment of couple due to shifting of centre of buoyancy from $B$ to $B_1$

$= F_B \times BB_1$

$= F_B \times BM \times \theta$ $\{\because BB_1 = BM \times \theta$ if $\theta$ is very small$\}$

$= W \times BM \times \theta$ $\{\because F_B = W\}$ ...(4.2)

But these two couples are the same. Hence equating equations (4.1) and (4.2), we get

$$W \times BM \times \theta = \int 2\rho g x^2 \theta \, Ldx$$

$$W \times BM \times \theta = 2\rho g \theta \int x^2 Ldx$$

$$W \times BM = 2\rho g \int x^2 Ldx$$

Now $Ldx$ = Elemental area on the water line shown in Fig. 4.6 ($c$) and = $dA$

$\therefore$ $$W \times BM = 2\rho g \int x^2 dA.$$

But from Fig. 4.5 ($c$) it is clear that $2 \int x^2 \, dA$ is the second moment of area of the plan of the body at water surface about the axis $Y$-$Y$. Therefore

$$W \times BM = \rho g I \qquad \{\text{where } I = 2 \int x^2 \, dA\}$$

$\therefore$ $$BM = \frac{\rho g I}{W}$$

But $W$ = Weight of the body

= Weight of the fluid displaced by the body

= $\rho g \times$ Volume of the fluid displaced by the body

= $\rho g \times$ Volume of the body sub-merged in water

= $\rho g \times \forall$

$\therefore$ $$BM = \frac{\rho g \times I}{\rho g \times \forall} = \frac{I}{\forall} \qquad \ldots(4.3)$$

$$GM = BM - BG = \frac{I}{\forall} - BG$$

$\therefore$ Meta-centric height $= GM = \frac{I}{\forall} - BG.$ $\qquad \ldots(4.4)$

**Problem 26:** *A rectangular pontoon is 5 m long, 3 m wide and 1.20 m high. The depth of immersion of the pontoon is 0.80 m in sea water. If the centre of gravity is 0.6 m above the bottom of the pontoon, determine the meta-centric height. The density for sea water = 1025 kg/m$^3$.*

Dimension of pontoon = 5 m × 3 m × 1.20 m

Depth of immersion = 0.8 m

Distance $AG$ = 0.6 m

Distance $AB = \frac{1}{2}$ × Depth of immersion

$= \frac{1}{2} \times .8 = 0.4$ m

Density for sea water = 1025 kg/m$^3$

Meta-centre height $GM$, given by equation (4.4) is

$$GM = \frac{I}{\forall} - BG$$

where $I$ = M.O. Inertia of the plan of the pontoon about $Y$-$Y$ axis

$$= \frac{1}{12} \times 5 \times 3^3 \text{ m}^4 = \frac{45}{4} \text{ m}^4$$

$\forall$ = Volume of the body sub-merged in water

$= 3 \times 0.8 \times 5.0 = 12.0$ m$^3$

$BG = AG - AB = 0.6 - 0.4 = 0.2$ m

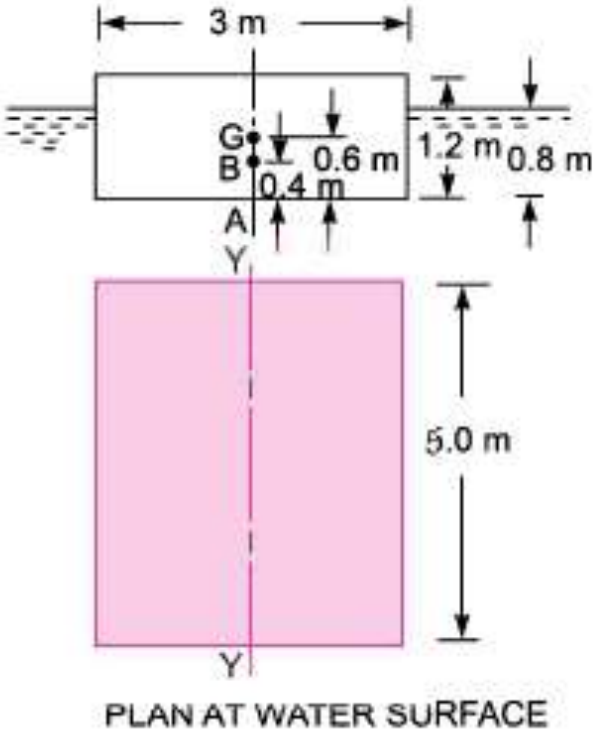



Fig. 4.7

$$\therefore \quad GM = \frac{45}{4} \times \frac{1}{12.0} - 0.2 = \frac{45}{48} - 0.2 = 0.9375 - 0.2 = \mathbf{0.7375\ m.}\ \textbf{Ans.}$$

**Problem 27:** *A block of wood of specific gravity 0.7 floats in water. Determine the meta-centric height of the block if its size is 2 m × 1 m × 0.8 m.*

Dimension of block = $2 \times 1 \times 0.8$

Let depth of immersion = $h$ m

Sp. gr. of wood = 0.7

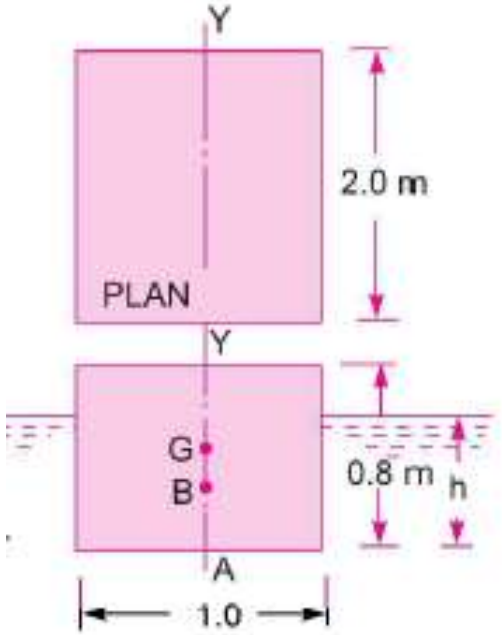
Weight of wooden piece = Weight density of wood* × Volume

$= 0.7 \times 1000 \times 9.81 \times 2 \times 1 \times 0.8$ N

Weight of water displaced = Weight density of water

× Volume of the wood sub-merged in water

$= 1000 \times 9.81 \times 2 \times 1 \times h$ N




For equilibrium,

Weight of wooden piece = Weight of water displaced

$\therefore \quad 700 \times 9.81 \times 2 \times 1 \times 0.8 = 1000 \times 9.81 \times 2 \times 1 \times h$

$\therefore \quad h = \dfrac{700 \times 9.81 \times 2 \times 1 \times 0.8}{1000 \times 9.81 \times 2 \times 1} = 0.7 \times 0.8 = 0.56$ m

$\therefore$ Distance of centre of Buoyancy from bottom, *i.e.*,

$$AB = \frac{h}{2} = \frac{0.56}{2} = 0.28 \text{ m}$$

and $AG = 0.8/2.0 = 0.4$ m

$\therefore \quad BG = AG - AB = 0.4 - 0.28 = 0.12$ m

The meta-centric height is given by equation (4.4) or

$$GM = \frac{I}{\forall} - BG$$

where $I = \dfrac{1}{12} \times 2 \times 1.0^3 = \dfrac{1}{6}$ m$^4$

$\forall$ = Volume of wood in water

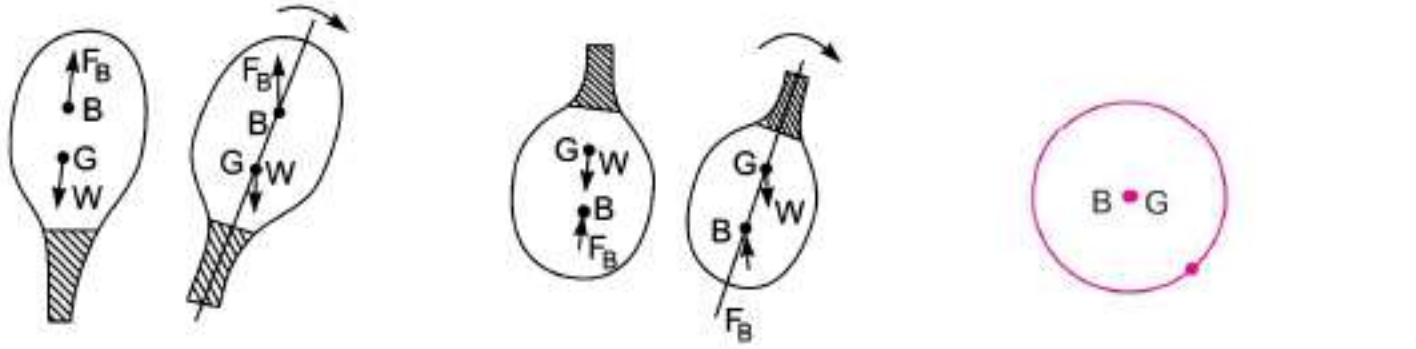
$= 2 \times 1 \times h = 2 \times 1 \times .56 = 1.12$ m$^3$

$\therefore \quad GM = \dfrac{1}{6} \times \dfrac{1}{1.12} - 0.12 = 0.1488 - 0.12 = \mathbf{0.0288}$ **m**

# CONDITIONS OF EQUILIBRIUM OF A FLOATING AND SUB-MERGED BODIES

A sub-merged or a floating body is said to be stable if it comes back to its original position after a slight disturbance. The relative position of the centre of gravity ($G$) and centre of buoyancy ($B_1$) of a body determines the stability of a sub-merged body.

**4.7.1 Stability of a Sub-merged Body.** The position of centre of gravity and centre of buoyancy in case of a completely sub-merged body are fixed. Consider a balloon, which is completely sub-merged in air. Let the lower portion of the balloon contains heavier material, so that its centre of gravity is lower than its centre of buoyancy as shown in Fig. 4.12 ($a$). Let the weight of the balloon is $W$. The weight $W$ is acting through $G$, vertically in the downward direction, while the buoyant force $F_B$ is acting vertically up, through $B$. For the equilibrium of the balloon $W = F_B$. If the balloon is given an angular displacement in the clockwise direction as shown in Fig. 4.12 ($a$), then $W$ and $F_B$ constitute a couple acting in the anti-clockwise direction and brings the balloon in the original position. Thus the balloon in the position, shown by Fig. 4.12 ($a$) is in stable equilibrium.




(a) STABLE EQUILIBRIUM (b) UNSTABLE EQUILIBRIUM (c) NEUTRAL EQUILIBRIUM

(*a*) **Stable Equilibrium.** When $W = F_B$ and point $B$ is above $G$, the body is said to be in stable equilibrium.

(*b*) **Unstable Equilibrium.** If $W = F_B$, but the centre of buoyancy ($B$) is below centre of gravity ($G$), the body is in unstable equilibrium as shown in Fig. 4.12 (*b*). A slight displacement to the body, in the clockwise direction, gives the couple due to $W$ and $F_B$ also in the clockwise direction. Thus the body does not return to its original position and hence the body is in unstable equilibrium.

(*c*) **Neutral Equilibrium.** If $F_B = W$ and $B$ and $G$ are at the same point, as shown in Fig. 4.12 (*c*), the body is said to be in neutral equilibrium.

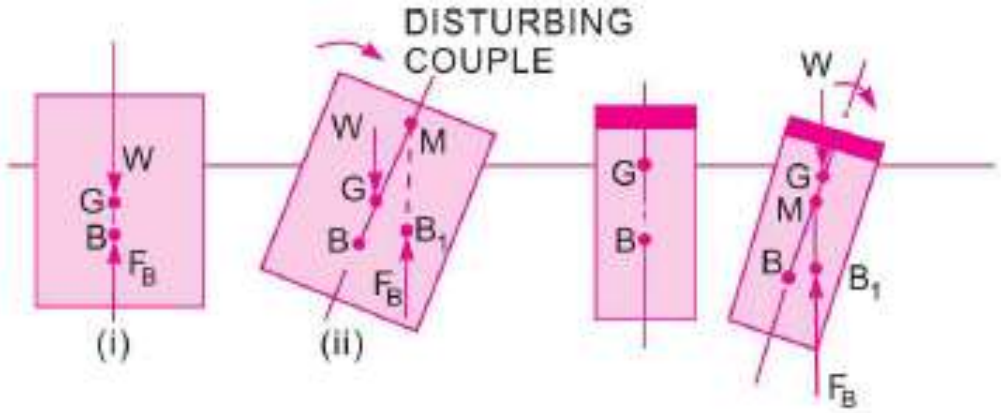
### 4.7.2 Stability of Floating Body.

The stability of a floating body is determined from the position of Meta-centre ($M$). In case of floating body, the weight of the body is equal to the weight of liquid displaced.

(*a*) **Stable Equilibrium.** If the point $M$ is above $G$, the floating body will be in stable equilibrium as shown in Fig. 4.13 (*a*). If a slight angular displacement is given to the floating body in the clockwise direction, the centre of buoyancy shifts from $B$ to $B_1$ such that the vertical line through $B_1$ cuts at $M$. Then the buoyant force $F_B$ through $B_1$ and weight $W$ through $G$ constitute a couple acting in the anti-clockwise direction and thus bringing the floating body in the original position.

(*b*) **Unstable Equilibrium.** If the point $M$ is below $G$, the floating body will be in unstable equilibrium as shown in Fig. 4.13 (*b*). The disturbing couple is acting in the clockwise direction. The couple due to buoyant force $F_B$ and $W$ is also acting in the clockwise direction and thus overturning the floating body.

(*c*) **Neutral Equilibrium.** If the point $M$ is at the centre of gravity of the body, the floating body will be in neutral equilibrium.




(a) Stable equilibrium $M$ is above $G$ (b) Unstable equilibrium $M$ is below $G$.

**Problem 28:** *A solid cylinder of diameter 4.0 m has a height of 4.0 m. Find the meta-centric height of the cylinder if the specific gravity of the material of cylinder = 0.6 and it is floating in water with its axis vertical. State whether the equilibrium is stable or unstable.*

**Solution.** Given : $D = 4$ m

Height, $h = 4$ m

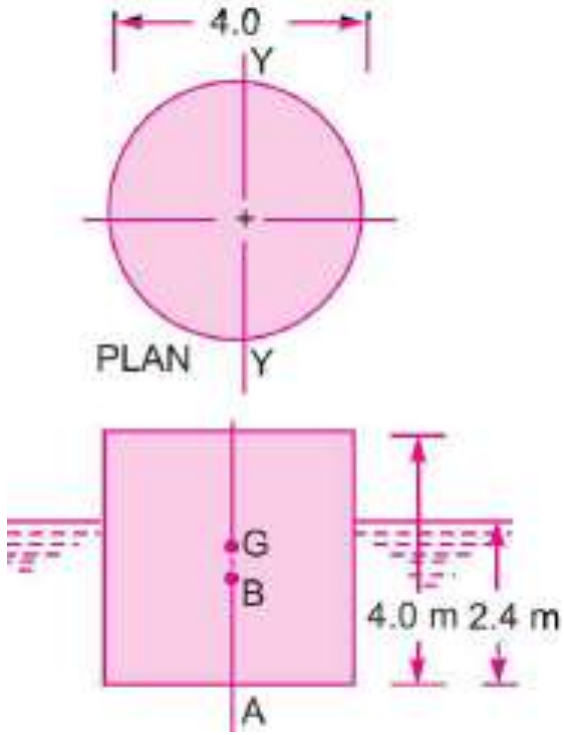
Sp. gr. $= 0.6$

Depth of cylinder in water = Sp. gr. $\times h$

$= 0.6 \times 4.0 = 2.4$ m

$\therefore$ Distance of centre of buoyancy ($B$) from $A$

$$AB = \frac{2.4}{2} = 1.2 \text{ m}$$



Distance of centre of gravity ($G$) from $A$

r $$AG = \frac{h}{2} = \frac{4.0}{2} = 2.0 \text{ m}$$

$\therefore$ $$BG = AG - AB = 2.0 - 1.2 = 0.8 \text{ m}$$

Now the meta-centric height $GM$ is given by

$$GM = \frac{I}{\forall} - BG$$

where $I$ = M.O.I. of the plan of the body about $Y$-$Y$ axis

$$= \frac{\pi}{64} D^4 = \frac{\pi}{64} \times (4.0)^4$$

$\forall$ = Volume of cylinder in water

$$= \frac{\pi}{4.0} \times D^2 \times \text{Depth of cylinder in water} = \frac{\pi}{4} \times 4^2 \times 2.4 \text{ m}^3$$

$\therefore$ $$\frac{I}{\forall} = \frac{\frac{\pi}{64} \times 4^4}{\frac{\pi}{4} \times 4^2 \times 2.4} = \frac{1}{16} \times \frac{4^2}{2.4} = \frac{1}{2.4} = 0.4167 \text{ m}$$

$\therefore$ $$GM = \frac{I}{\forall} - BG = 0.4167 - 0.8 = \mathbf{-0.3833 \text{ m. Ans.}}$$

–ve sign means that the meta-centre ($M$) is below the centre of gravity ($G$). Thus the cylinder is in unstable equilibrium. **Ans.**

**Problem 29:** *A solid cylinder of diameter 4.0 m has a height of 3 metres. Find the meta-centric height of the cylinder when it is floating in water with its axis vertical. The sp. gr. of the cylinder = 0.6.*

**Solution.** Given :

Dia. of cylinder, $D = 4.0$ m

Height of cylinder, $h = 3.0$ m

Sp. gr. of cylinder = 0.6

Depth of immersion of cylinder

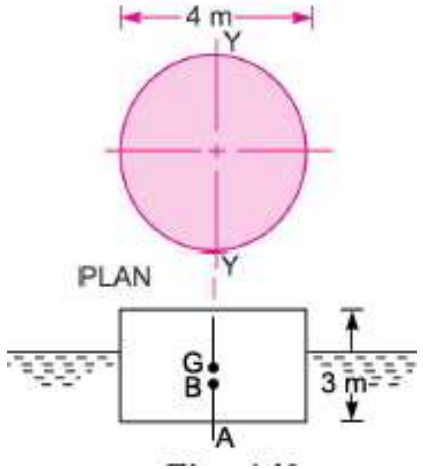
$= 0.6 \times 3.0 = 1.8$ m

$\therefore \quad AB = \frac{1.8}{2} = 0.9$ m

and $\quad AG = \frac{3}{2} = 1.5$ m

$\therefore \quad BG = AG - AB$

$= 1.5 - 0.9 = 0.6$ m




Now the meta-centric height $GM$ is given by equation (4.4)

$$GM = \frac{I}{\forall} - BG$$

But $I$ = M.O.I. about $Y$-$Y$ axis of the plan of the body

$$= \frac{\pi}{64} D^4 = \frac{\pi}{64} \times (4.0)^4$$

$\forall$ = Volume of cylinder in water

$$= \frac{\pi}{4} D^2 \times \text{Depth of immersion}$$

$$= \frac{\pi}{4} (4)^2 \times 1.8 \text{ m}^3$$

$\therefore$

$$GM = \frac{\frac{\pi}{64} \times (4.0)^4}{\frac{\pi}{4} \times (4.0)^2 \times 1.8} - 0.6$$

$$= \frac{1}{16} \times \frac{4.0^2}{1.8} - 0.6 = \frac{1}{1.8} - 0.6 = 0.55 - 0.6 = \mathbf{-0.05 \text{ m.}}$$

– ve sign means that meta-centre, ($M$) is below the centre of gravity ($G$).