

Unit 2: CW AND FREQUENCY MODULATED RADAR

Doppler Effect:

A technique for separating the received signal from the transmitted signal when there is relative motion between radar and target is based on recognizing the change in the echo-signal frequency caused by the Doppler effect.

It is well known in the fields of optics and acoustics that if either the source of oscillation or the observer of the oscillation is in motion, an apparent shift in frequency will result. This is the Doppler effect and is the basis of CW radar. If R is the distance from the radar to target, the total number of wavelengths λ contained in the two-way path between the radar and the target are $2R/\lambda$. The distance R and the wavelength λ are assumed to be measured in the same units. Since one wavelength corresponds to an phase angle excursion of 2π radians, the total phase angle excursion ϕ made by the electromagnetic wave during its transit to and from the target is $4\pi R/\lambda$ radians. If the target is in motion, R and the phase ϕ are continually changing. A change in ϕ with respect to time is equal to frequency. This is the Doppler angular frequency ω_d and is given by:

$$\omega_d = 2\pi f_d = d\phi/dt = d(4\pi R/\lambda)/dt = (4\pi/\lambda) \cdot dR/dt = (4\pi/\lambda) \cdot V_r = 4\pi \cdot V_r / \lambda$$

where f_d is the Doppler frequency shift in Hz, and V_r = relative velocity of the target with respect to the Radar. The Doppler frequency shift f_d is given by

$$f_d = 2V_r / \lambda = 2V_r f_0 / c$$

where f_0 is the transmitted frequency and c is the velocity of propagation of the electromagnetic waves (same as that of light) = 3×10^8 m/s. If f_d is in hertz, V_r in knots, and λ in meters then the Doppler frequency f_d is given by

$$f_d = 1.03 V_r / \lambda$$

A plot of this equation is shown in the figure below

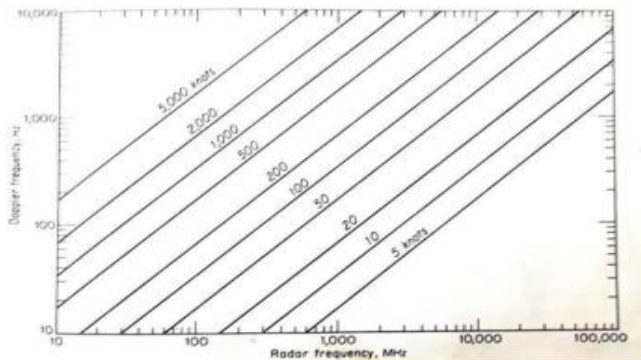


Figure: Doppler frequency f_d as a function of radar frequency and target relative velocity.

The relative velocity may be written as $V_r = V \cdot \cos \theta$ where V is the target speed and θ is angle made by the target trajectory and the line joining radar and target. When $\theta=0$ the Doppler frequency is maximum. The Doppler is zero when the trajectory is perpendicular to the radar line of sight ($\theta=90^\circ$).

The CW radar is of interest not only because of its many applications, but its study also serves as a means for better understanding the nature and use of the Doppler information contained in the echo signal, whether in a CW or a pulse radar (MTI) application. In addition to allowing the received signal to be separated from the transmitted signal, the CW radar provides a measurement of relative velocity which may be used to distinguish moving targets from stationary objects or clutter.

CW radar:

Consider the simple CW radar as illustrated by the block diagram of Figure below.

- The transmitter generates a continuous (unmodulated) oscillation of frequency f_0 , which is radiated by the antenna. A portion of the radiated energy is intercepted by the target and is scattered, some of it in the direction of the radar, where it is collected by the receiving antenna.
- If the target is in motion with a velocity V_r relative to the radar, the received signal will be shifted in frequency from the transmitted frequency f_0 by an amount $\pm f_d$ as given by the equation :
$$f_d = \frac{2V_r}{\lambda} = \frac{2V_r f_0}{c}$$
- The plus sign associated with the Doppler frequency applies if the distance between target and radar is decreasing (approaching target) that is, when the received signal frequency is greater than the transmitted signal frequency.
- The minus sign applies if the distance is increasing (receding target).
- The received echo signal at a frequency $f_0 \pm f_d$ enters the radar via the antenna and is heterodyned in the detector (mixer) with a portion of the transmitter signal f_0 to produce a Doppler beat note of frequency f_d .
- The sign of f_d is lost in this process. The purpose of the Doppler amplifier is to eliminate echoes from stationary targets and to amplify the Doppler echo signal to a level where it can operate an indicating device.
- It's frequency response characteristic is shown in the figure (b) below. The low-frequency cutoff must be high enough to reject the d-c component caused by stationary targets, but yet it must be low enough to pass the smallest Doppler frequency expected.
- Sometimes both conditions cannot be met simultaneously and a compromise is necessary. The upper cutoff frequency is selected to pass the highest Doppler frequency expected.

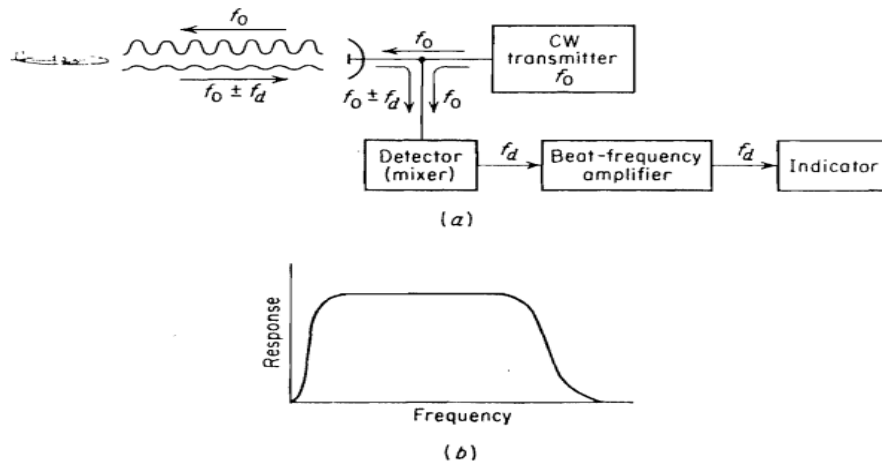


Figure : Simple CW radar block diagram (b) response characteristic of beat-frequency amplifier

Isolation between transmitter and receiver:

- In principle, a single antenna may be employed since the necessary isolation between the transmitted and the received signals is achieved via separation in frequency as a result of the Doppler Effect.
- In practice, it is not possible to eliminate completely the transmitter leakage. However, transmitter leakage is neither always undesirable.
- A moderate amount of leakage entering the receiver along with the echo signal supplies the reference necessary for the detection of the Doppler frequency shift. If a leakage signal of sufficient magnitude were not present, a sample of the transmitted signal has to be deliberately introduced into the receiver to provide the necessary reference frequency.
- There are two practical effects which limit the amount of transmitter leakage power which can be tolerated at the receiver. These are:
 - (1) The maximum amount of power the receiver input circuitry can withstand before it is physically damaged or its sensitivity reduced (burnout) and
 - (2) The amount of transmitter noise due to hum, microphonics, stray pick-up & instability which enters the receiver from the transmitter and affects the receiver sensitivity.

Hence additional isolation is usually required between the transmitter and the receiver if the sensitivity is not to be degraded either by burnout or by excessive noise. The amount of isolation required depends on the transmitter power and the accompanying transmitter noise as well as the ruggedness and the sensitivity of the receiver. For example, If the safe value of power which might be applied to a receiver is 10 mW and if the transmitter power is 1 kW, the isolation between transmitter and receiver must be at least 50 dB.

- The amount of isolation needed in a long-range CW radar is more often determined by the noise that accompanies the transmitter leakage signal rather than by any damage caused by high power. For example, suppose the isolation between the transmitter and receiver is such that 10 mW of leakage signal appeared at the receiver. If the minimum detectable signal is 10-13 watt (100 dB below 1mW), the transmitter noise must be at least 110 dB (preferably 120 or 130 dB) below the transmitted carrier.

- The transmitter noise of concern in Doppler radar includes those noise components that lie within the same frequency range as the Doppler frequencies. If complete elimination of the direct leakage signal at the receiver could be achieved, it might not entirely solve the isolation problem since echoes from nearby fixed targets (clutter) can also contain the noise components of the transmitted signal.
- The receiver of a pulsed radar is isolated and protected from the damaging effects of the transmitted pulse by the duplexer, which short-circuits the receiver input during the transmission period. Turning off the receiver during transmission with a duplexer is not possible in a CW radar since the transmitter is operated continuously.
- In CW Radars Isolation between transmitter and receiver might be obtained with a single antenna by using a hybrid junction, circulator, turnstile junction, or with separate polarizations. Separate antennas for transmitting and receiving might also be used.
 - The amount of isolation which can be readily achieved between the arms of practical hybrid junctions such as the magic-T, rat race, or short-slot coupler is of the order of 20 to 30 dB. In some instances, when extreme precision is exercised, an isolation of perhaps 60 dB or more might be achieved. But one limitation of the hybrid junction is the 6-dB loss in overall performance which results from the inherent waste of half the transmitted power and half the received signal power. Both the loss in performance and the difficulty in obtaining large isolations have limited the application of the hybrid junction to short-range radars.
 - Ferrite isolation devices such as the circulator do not suffer the 6-dB loss inherent in the hybrid junction. Practical devices have isolation of the order of 20 to 50 dB. Turnstile junctions achieve isolations as high as 40 to 60 dB.
 - The use of orthogonal polarizations for transmitting and receiving is limited to short range radars because of the relatively small amount of isolation that can be obtained.
- An important factor which limits the use of isolation devices with a common antenna is the reflections produced in the transmission line by the antenna. The reflection coefficient from a mismatched antenna with a voltage standing-wave ratio ζ is $|\rho| = (\zeta - 1) / (\zeta + 1)$. Therefore, if an isolation of 20 dB is to be obtained, the VSWR must be less than 1.22. If 40 dB of isolation is required, the VSWR must be less than 1.02.
- The largest isolations are obtained with two antennas: one for transmission, the other for reception-physically separated from one another. Isolations of the order of 80 dB or more are possible with high-gain antennas. The more directive the antenna beam and the greater the spacing between antennas, the greater will be the isolation. A common radome enclosing the two antennas should be avoided since it limits the amount of isolation that can be achieved.
- Additional isolation can be obtained by properly introducing a controlled sample of the transmitted signal directly into the receiver. The phase and amplitude of this “buck-off” signal are adjusted to cancel the portion of the transmitter signal that leaks into the receiver. An additional 10 dB of isolation might be obtained.
- The transmitter signal is never a pure CW waveform. Minute variations in amplitude (AM) and phase (FM) can result in sideband components that fall within the Doppler frequency band. These can generate false targets or mask the desired signals. Therefore, both AM and FM modulations can result in undesired sidebands. AM sidebands are typically 120 dB below the carrier, as measured in a 1 kHz band, and are relatively constant across the usual Doppler spectrum of interest. The normal antenna isolation plus “feed through nulling” usually reduces the AM components below receiver noise in

moderate power radars. FM sidebands are usually significantly greater than AM, but decrease with increasing offset from the carrier. These can be avoided by stabilizing the output frequency of the CW transmitter and by feeding back the extracted FM noise components so as to reduce the original frequency deviation.

Intermediate-frequency receiver:

Limitation of Zero IF receiver:

- The receiver in the simple CW radar shown earlier is in some respects analogous to a super heterodyne receiver. Receivers of this type are called homodyne receivers, or super heterodyne receivers with zero IF.
- The function of the local oscillator is replaced by the leakage signal from the transmitter. Such a receiver is simpler than the one with a more conventional intermediate frequency since no IF amplifier or local oscillator is required.
- However, this simpler receiver is not very sensitive because of increased noise at the lower intermediate frequencies caused by flicker effect.
- Flicker-effect noise occurs in semiconductor devices such as diode detectors and cathodes of vacuum tubes. The noise power produced by the flicker effect varies as $1/f^\alpha$ where α is approximately unity.
- This is in contrast to shot noise or thermal noise, which is independent of frequency. Thus, at the lower range of frequencies (audio or video region), where the Doppler frequencies usually are found, the detector of the CW receiver can introduce a considerable amount of flicker noise, resulting in reduced receiver sensitivity.
- For short-range, low-power, applications this decrease in sensitivity might be tolerated since it can be compensated by a modest increase in antenna aperture and/or additional transmitter power.
- But for maximum efficiency with CW radar, the reduction in sensitivity caused by the simple Doppler receiver with zero IF cannot be tolerated.

Non zero IF Receiver:

- The effects of flicker noise are overcome in the normal super heterodyne receiver by using an intermediate frequency high enough to make the flicker noise small compared with the normal receiver noise. This results from the inverse frequency dependence of flicker noise.
- Figure below shows the block diagram of a CW radar whose receiver operates with a nonzero IF. Separate antennas are shown for transmission and reception.
- Instead of the usual local oscillator found in the conventional super heterodyne receiver, the local oscillator (or reference signal) is derived in the receiver from a portion of the transmitted signal mixed with a locally generated signal of frequency equal to that of the receiver IF.
- Since the output of the mixer consists of two sidebands on either side of the carrier plus higher harmonics, a narrowband filter selects one of the sidebands as the reference signal.
- The improvement in receiver sensitivity with an intermediate-frequency super heterodyne might be as much as 30 dB over the simple zero IF receiver.

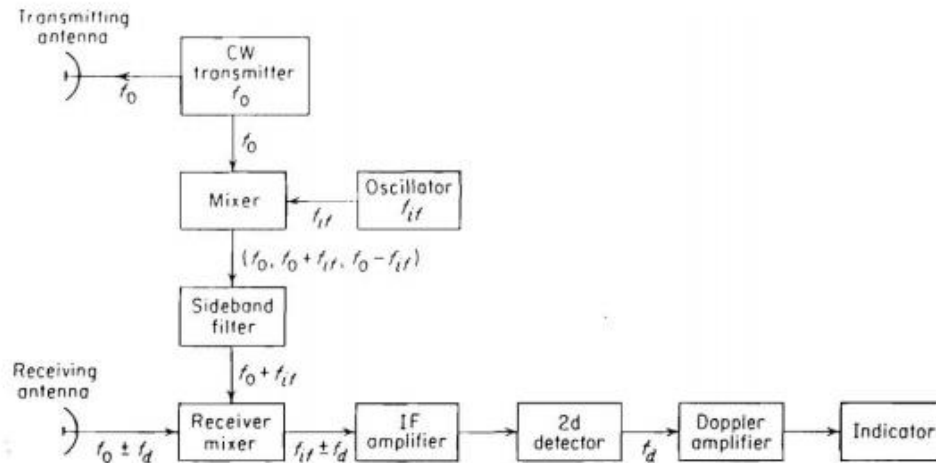


Figure: Block diagram of a CW Doppler radar with nonzero IF receiver, also called sideband super heterodyne Receiver.

Receiver bandwidth requirements:

- One of the requirements of the Doppler-frequency amplifier in the simple CW radar (Zero IF) or the IF amplifier of the sideband super heterodyne (Non Zero IF) is that it has to be wide enough to pass the expected range of Doppler frequencies.
- In most cases of practical interest the expected range of Doppler frequencies will be much wider than the frequency spectrum occupied by the signal energy. Consequently, the use of a wideband amplifier covering the expected Doppler range will result in an increase in noise and a lowering of the receiver sensitivity.
- If the frequency of the Doppler-shifted echo signal were known beforehand, narrowband filter—that is just wide enough to reduce the excess noise without eliminating a significant amount of signal energy might be used.
- If the waveforms of the echo signal are known, as well as its carrier frequency, a matched filter could also be considered. Several factors tend to spread the CW signal energy over a finite frequency band. These must be known if an approximation to the bandwidth required for the narrowband Doppler filter is to be obtained.
- If the received waveform were a sine wave of infinite duration, its frequency spectrum would be a delta function as shown in the figure (a) below and the receiver bandwidth would be infinitesimal. But a sine wave of infinite duration and an infinitesimal bandwidth cannot occur in nature. The more normal situation is an echo signal which is a sine wave of finite duration.
- The frequency spectrum of a finite-duration sine wave has a shape of the form $[\sin\pi(f-f_0)\delta]/\pi(f-f_0)$ where f_0 and δ are the frequency and duration of the sine wave, respectively, and f is the frequency variable over which the spectrum is plotted (Fig b).

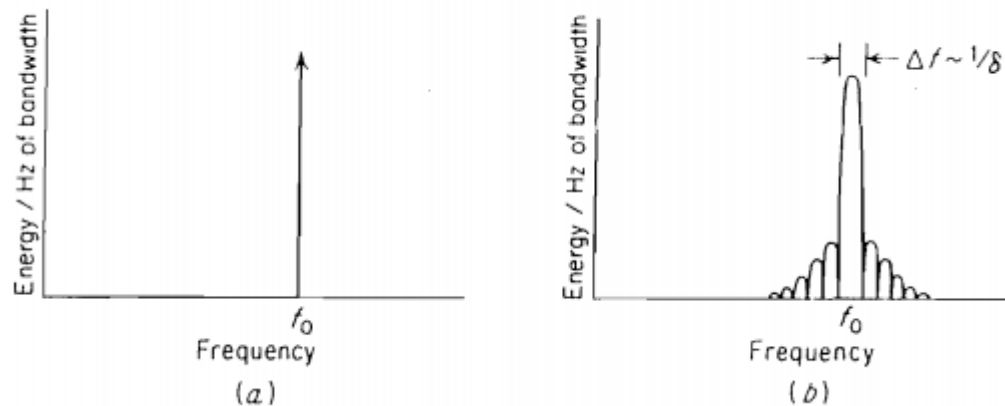


Figure: Frequency spectrum of CW oscillation of (a) infinite duration and (b) finite duration

Applications of CW radar:

- Measurement of the relative velocity of a moving target, as in the police speed monitor or in the rate-of-climb meter for vertical-take-off aircraft.
- Control of traffic lights, regulation of tollbooths, vehicle counting.
- As a sensor in antilock braking systems, and for collision avoidance.
- In railways, as a speedometer to replace the conventional axle-driven tachometer. In such an application it would be unaffected by errors caused by wheel slip on accelerating or wheel slide when braking.
- Monitoring the docking speed of large ships.
- Measurement of the velocity of missiles, ammunition, and baseballs.

Advantages and disadvantages of CW Radars:

- The principal advantage of CW Doppler radar over the other (non radar) methods of measuring speed is that there need not be any physical contact with the object whose speed is being measured. In industry this is used to measure turbine-blade vibration, the peripheral speed of grinding wheels, and the monitoring of vibrations in the cables of suspension bridges.
- Most of the above applications can be satisfied with a simple, solid-state CW source with powers in tens of milli watts .
- High-power CW radars for the detection of aircraft and other targets have been developed and have been used in such systems as the Hawk missile systems. (Shown below) .
- The difficulty of eliminating the leakage of the transmitter signals into the receiver has limited the utility of unmodulated CW radar for many long-range applications.
- The CW radar, when used for short or moderate ranges, is characterized by simpler equipment than a pulse radar. The amount of power that can be used with a CW radar is dependent on the isolation that can be achieved between the transmitter and receiver since the transmitter noise that finds its way into the receiver limits the receiver sensitivity. (The pulse radar has no similar limitation to its maximum range because the transmitter is not operative when the receiver is turned on.)

- Major disadvantage of the simple CW radar is its inability to obtain a measurement of range. This limitation can be overcome by modulating the CW carrier, as in the frequency-modulated radar.
- Some anti-air-warfare guided missile systems employ semi active homing guidance in which a receiver in the missile receives energy from the target, the energy having been transmitted from an "illuminator" external to the missile. The illuminator will be at the launch platform. CW illumination has been used in many successful systems. An example is the Hawk tracking illuminator shown in the figure below. It is tracking radar as well as an illuminator since it must be able to follow the target as it travels through space.

FM-CW RADAR

Introduction:

- The inability of the simple CW radar to measure range is mainly due to the lack of a Timing mark. The timing mark permits the time of transmission and the time of return to be recognized but it increases the spectrum of the transmitted waveform.
- The sharper or more distinct the mark, the more accurate the measurement of the transit time. But the more distinct the timing mark, the broader will be the transmitted spectrum.
- This follows from the properties of the Fourier transform. Therefore a finite spectrum of necessity must be transmitted if transit time or range is to be measured.
- The spectrum of a CW transmission can be broadened by the application of a modulation - amplitude, frequency, or phase.
- An example of an amplitude modulation is the pulse radar. A widely used technique to insert a timing mark is to frequency modulate the carrier.
- The timing mark is the changing frequency. The transit time is proportional to the difference in frequency between the transmitter signal and the echo signal.
- The greater the transmitter frequency deviation in a given time interval, the more accurate is the measurement of the transit time but the transmitted spectrum also becomes larger.

Range and Doppler measurement:

In the frequency-modulated CW radar (abbreviated FM-CW), the transmitter frequency is changed as a function of time in a known manner. Assume that the transmitter frequency increases linearly with time, as shown by the solid line in the figure below.

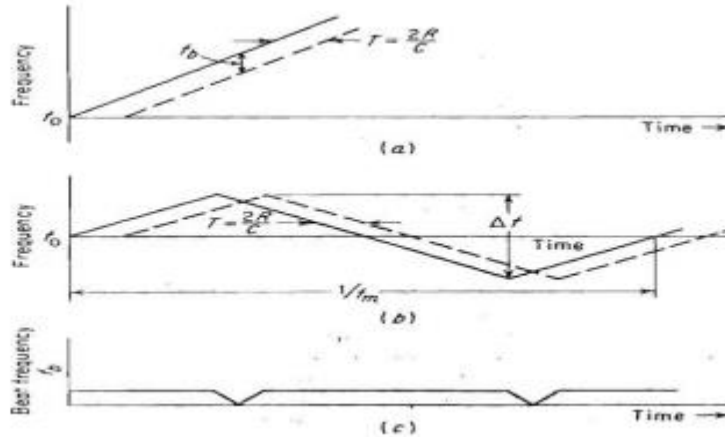


Figure: Frequency-time relationships in FM-CW radar. Solid curve represents transmitted signal; dashed curve represents echo. (a) Linear frequency modulation (b) triangular frequency modulation(c) beat note of (b)

If there is a reflecting object at a distance R , the echo signal will return after a time $T = 2R/c$. The dashed line in the figure represents the echo signal. When the echo signal is heterodyned with a portion of the transmitter signal in a nonlinear element such as a diode, a beat note f_b will be produced. If there is no Doppler frequency shift, the beat note (difference frequency) is a measure of the target's range and $f_b = f_r$ where f_r is the beat frequency only due to the target's range. If the rate of change of the carrier frequency is $f_0(\dot{})$ then the beat frequency is given by:

$$f_r = \dot{f}_0 T = \frac{2R}{c} \dot{f}_0$$

In any practical CW radar, the frequency cannot be continually changed in one direction only. Periodicity in the modulation is necessary, as in the triangular frequency-modulation waveform shown in fig.b. The modulation need not necessarily be triangular. It can be saw tooth, sinusoidal, or some other shape. The resulting beat frequency as a function of time is shown in fig.c for triangular modulation. The beat note is of constant frequency except at the turnaround region. If a frequency change of Δf is modulated at a rate f_m , then the beat frequency is

$$f_r = (2R/c) \cdot 2f_m \cdot \Delta f = 4Rf_m \cdot \Delta f / c$$

Or $R = c f_r / 4f_m \cdot \Delta f \dots \dots \dots [Eq.1]$

Thus the measurement of the beat frequency determines the range R . A block diagram illustrating the principle of the FM-CW radar is shown in the figure below. A portion of the transmitter signal acts as the reference signal required to produce the beat frequency. It is introduced directly into the receiver via a cable or other direct connection.

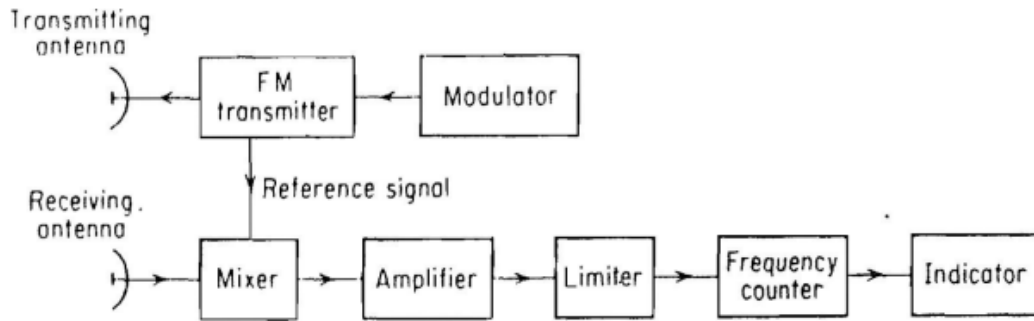


Figure: Block diagram of FM-CW radar

Ideally the isolation between transmitting and receiving antennas is made sufficiently large so as to reduce to a negligible level the transmitter leakage signal which arrives at the receiver via the coupling between antennas. The beat frequency is amplified and limited to remove any amplitude fluctuations. The frequency of the amplitude-limited beat note is measured with a cycle-counting frequency meter calibrated in distance.

In the above, the target was assumed to be stationary. If this assumption is not applicable, a Doppler frequency shift will be superimposed on the FM range beat note and an erroneous range measurement results. The Doppler frequency shift causes the frequency-time plot of the echo signal to be shifted up or down as shown in the figure (a). On one portion of the frequency-modulation cycle the beat frequency (fig. b) is increased by the Doppler shift, while on the other portion, it is decreased.

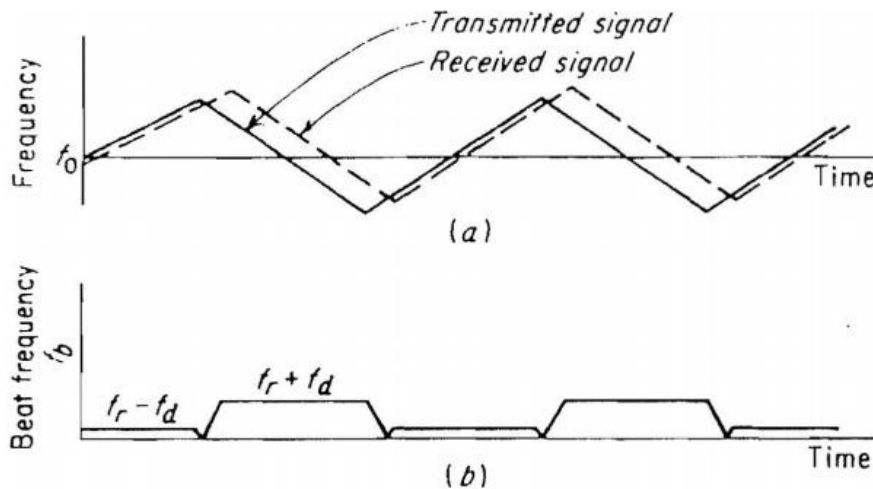


Figure: Frequency-time relationships in FM-CW radar when the received signal is shifted in frequency by the Doppler effect (a) Transmitted (solid curve) and echo (dashed curve) (b) beat frequency

If for example, the target is approaching the radar, the beat frequency f_b (up) produced during the increasing or up portion of the FM cycle will be the difference between the beat frequency due

to the range f_r and the Doppler frequency shift f_d . Similarly, on the decreasing portion, the beat frequency f_b (down) is the sum of the two.

$$f_b(\text{up}) = f_r - f_d$$

$$f_b(\text{down}) = f_r + f_d$$

The range frequency f_r may be extracted by measuring the average beat frequency; that is, $\frac{1}{2}[f_b(\text{up}) + f_b(\text{down})] = f_r$. If $f_b(\text{up})$ and $f_b(\text{down})$ are measured separately, for example, by switching a frequency counter every half modulation cycle, one-half the difference between the frequencies will yield the Doppler frequency. This assumes $f_r > f_d$. If, on the other hand, $f_r < f_d$ such as might occur with a high-speed target at short range, the roles of the averaging and the difference-frequency measurements are reversed; the averaging meter will measure Doppler velocities, and the difference meter measures range.

FM-CW Altimeter:

The FM-CW radar principle is used in the aircraft radio altimeter to measure height above the surface of the earth. The large backscatter cross section and the relatively short ranges required of altimeters permit low transmitter power and low antenna gain. Since the relative motion between the aircraft and ground is small, the effect of the Doppler frequency shift also may usually be neglected. The band from 4.2 to 4.4GHz is reserved for radio altimeters, although they have in the past operated at UHF. The transmitter power is relatively low and can be obtained from a CW Magnetron, a backwardwave oscillator, or a reflex klystron, but now they have been replaced by the solid state transmitter. The altimeter can employ a simple homodyne receiver, but for better sensitivity and stability the super heterodyne is preferred whenever its more complex construction can be tolerated. The block diagram of the FM-CW radar with a sideband super heterodyne receiver is shown in the figure below.

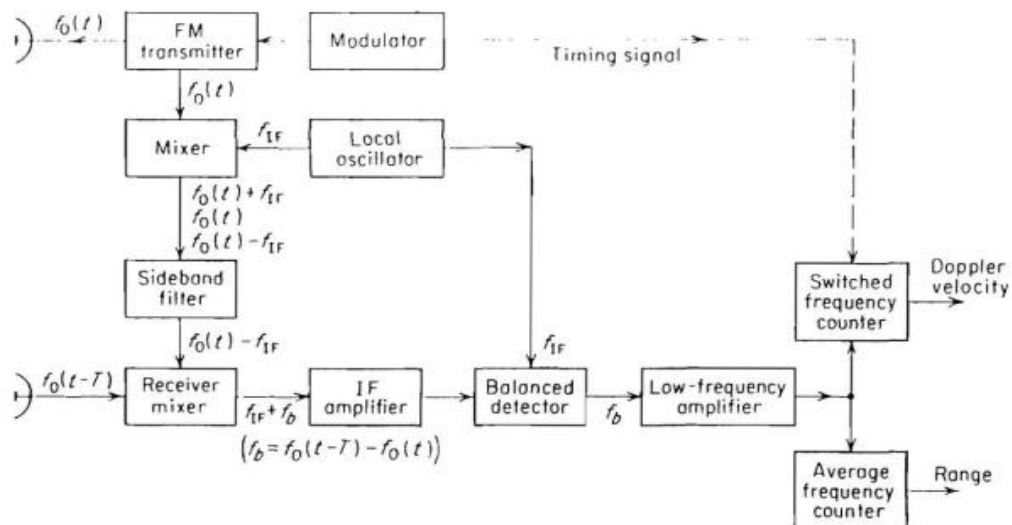


Figure: Block diagram of a FM-CW radar using sideband super heterodyne receiver

A portion of the frequency-modulated transmitted signal is applied to a mixer along with the oscillator signal. The selection of the local-oscillator frequency is a bit different from that in the usual super heterodyne receiver. The local-oscillator frequency f_{IF} is the same as the intermediate

frequency used in the receiver, whereas in the conventional super heterodyne the LO frequency is of the same order of frequency as the RF signal.

The output of the mixer consists of the varying transmitter frequency $f_o(t)$ plus two sideband frequencies, one on either side of $f_o(t)$ and separated from $f_o(t)$ by the local-oscillator frequency f_{IF} . The filter selects the lower sideband, $f_o(t) - f_{IF}$ and rejects the carrier and the upper sideband. The side band that is passed by the filter is modulated in the same fashion as the transmitted signal. The sideband filter must have sufficient bandwidth to pass the modulation, but not the carrier or other sideband. The filtered sideband serves the function of the local oscillator.

When an echo signal is present, the output of the receiver mixer is an IF signal of frequency $(f_{IF} + f_b)$ where f_b is composed of the range frequency f_r and the Doppler velocity frequency f_d . The IF signal is amplified and applied to the balanced detector along with the local-oscillator signal f_{IF} . The output of the detector contains the beat frequency (range frequency and the Doppler velocity frequency), which is amplified to a level where it can actuate the frequency-measuring circuits.

In the above figure, the output of the low-frequency amplifier is divided into two channels: one feeds an average-frequency counter to determine the range, and the other feeds a switched frequency counter to determine the Doppler velocity (assuming $f_r > f_d$). Only the averaging frequency counter need be used in an altimeter application, since the rate of change of altitude is usually small.

A target at short range will generally result in a strong signal at low frequency, while one at long range will result in a weak signal at high frequency. Therefore the frequency characteristic of the low frequency amplifier in the FM-CW radar may be shaped to provide attenuation at the low frequencies corresponding to short ranges and large echo signals. Less attenuation is applied to the higher frequencies, where the echo signals are weaker.

Multiple-frequency CW Radar:

Consider a CW radar radiating a single-frequency sine wave of the form $\sin 2\pi f_o t$ (The amplitude of the signal is taken to be unity since it does not influence the result) the signal travels to the target at a range R and returns to the radar after a time $T = 2R/c$ where c is the velocity of propagation. The echo signal received at the radar is $\sin [2\pi f_o(t - T)]$. If the transmitted and received signals are compared in a phase detector, the output is proportional to the phase difference between the two and is given by :

$$\Delta\phi = 2\pi f_o T = 4\pi f_o R/c.$$

The phase difference may therefore be used as a measure of the range, or

$$R = \frac{c \Delta\phi}{4\pi f_o} = \frac{\lambda}{4\pi} \Delta\phi \quad \dots\dots\dots \text{[Eq. 2]}$$

However, the measurement of the phase difference $\Delta\phi$ is unambiguous only if $\Delta\phi$ does not exceed 2π radians. Substituting $\Delta\phi = 2\pi$ into the above equation (Eq.1) gives the maximum unambiguous range as $\lambda/2$. At radar frequencies this unambiguous range is much too small to be of any practical interest.

Unambiguous range may be extended considerably by utilizing two separate CW signals differing slightly in frequency. The unambiguous range in this case corresponds to half wavelength at the difference frequency.

The transmitted waveform is assumed to consist of two continuous sine waves of frequency f_1 and f_2 separated by an amount Δf . For convenience, the amplitudes of all signals are set equal to unity. The voltage waveforms of the two components of the transmitted signal v_{1T} and v_{2T} may be written as

$$v_{1T} = \sin(2\pi f_1 t + \phi_1)$$

$$v_{2T} = \sin(2\pi f_2 t + \phi_2)$$

where ϕ_1 and ϕ_2 are arbitrary (constant) phase angles. The echo signal is shifted in frequency by the Doppler Effect. The form of the Doppler-shifted signals corresponding to the two frequencies f_1 and f_2 are:

$$v_{1R} = \sin \left[2\pi(f_1 \pm f_{d1})t - \frac{4\pi f_1 R_0}{c} + \phi_1 \right]$$

$$v_{2R} = \sin \left[2\pi(f_2 \pm f_{d2})t - \frac{4\pi f_2 R_0}{c} + \phi_2 \right]$$

Where R_0 = range to target at a particular time

t = t_0 (range that would be measured if target were not moving)

f_{d1} = Doppler frequency shift associated with frequency f_1

f_{d2} = Doppler frequency shift associated with frequency f_2

Since the two RF frequencies f_1 and f_2 are approximately the same (that is $f_2 = f_1 + \Delta f$, where $\Delta f \ll f_1$) the Doppler frequency shifts f_{d1} and f_{d2} can be assumed to be equal to each other. Therefore we may write $f_{d1} = f_{d2} = f_d$.

The receiver separates the two components of the echo signal and heterodynes each received signal component with the corresponding transmitted waveform and extracts the two Doppler-frequency components given below:

$$v_{1D} = \sin \left(\pm 2\pi f_d t - \frac{4\pi f_1 R_0}{c} \right)$$

$$v_{2D} = \sin \left(\pm 2\pi f_d t - \frac{4\pi f_2 R_0}{c} \right)$$

The phase difference between these two components is

$$\Delta\phi = \frac{4\pi(f_2 - f_1)R_0}{c} = \frac{4\pi \Delta f R_0}{c}$$

Hence

$$R_0 = \frac{c \Delta\phi}{4\pi \Delta f}$$

which is same as that of Eq..2, with Δf substituted in place of f_0 .

Important aspects of Multi Frequency Radar:

- The two frequencies of the two-frequency radar were described as being transmitted simultaneously. They may also be transmitted sequentially in some applications by rapidly switching a single RF source.
- A large difference in frequency between the two transmitted signals improves the accuracy of the range measurement since large Δf means a proportionately large change in $\Delta\theta$ for a given range. However, there is a limit to the value of Δf since $\Delta\theta$ cannot be greater than 2π radians if the range is to remain unambiguous. The maximum unambiguous range R_{unamb} is

$$R_{unamb} = \frac{c}{2 \Delta f}$$

Therefore Δf must be less than $c/2R_{unamb}$. Note that when Δf is replaced by the pulse repetition rate, the above equation gives the maximum unambiguous range of a pulse radar.

- A qualitative explanation of the operation of the two-frequency radar may be had by considering both carrier frequencies to be in phase at zero range. As they progress outward from the radar, the relative phase between the two increases because of their difference in frequency. This phase difference may be used as a measure of the elapsed time. When the two signals slip in phase by one cycle, the measurement of phase, and hence range, becomes ambiguous.
- The two-frequency CW radar is essentially a single-target radar since only one phase difference can be measured at a time. If more than one target is present, the echo signal becomes complicated and the meaning of the phase measurement becomes doubtful.
- The theoretical rms range error with which range can be measured with the two-frequency CW radar was estimated to be

$$\delta R = \frac{c}{4\pi \Delta f (2E/N_0)^{1/2}}$$

Where E = energy contained in received signal and N_0 = noise power per hertz of bandwidth. The above Equation indicates that the greater the separation Δf between the two frequencies, the lesser will be the rms error.

- However if the frequency difference Δf increases unambiguous Range decreases. The selection of Δf represents a compromise between the requirements of accuracy and ambiguity. Both accurate and unambiguous range measurements can be made by transmitting three or more frequencies instead of just two.
- For example, if the three frequencies f_1, f_2 and f_3 are such that $f_3 - f_1 = k(f_2 - f_1)$ where k is a factor of the order of 10 or 20, the pair of frequencies f_3, f_1 (with greater Δf) gives an ambiguous but accurate range measurement while the pair of frequencies f_2, f_1 (with lesser Δf) resolve the ambiguities in the measurement of Range. Likewise, further accuracy improvement without reducing the ambiguous range can be obtained by adding more frequencies. As more frequencies are added the spectrum and target resolution approach that obtained with a pulse or an FM-CW waveform.

Illustrative problems:

1. Determine the Range and Doppler velocity of an approaching target using a triangular modulation FMCW Radar. Given : Beat frequency fb(up) = 15KHz and fb (down) = 25KHz , modulating frequency : 1MHz, Δf : 1KHz and Operating frequency : 3Ghz.

Solution:

$$\text{We know } f_r = \frac{1}{2}[\text{fb}(\text{up}) + \text{fb}(\text{down})] = \frac{1}{2}(15+25) = 20 \text{ KHz}$$

$$f_d = \frac{1}{2}[\text{fb}(\text{down}) - \text{fb}(\text{up})] = \frac{1}{2}(25-15) = 5 \text{ KHz}$$

The Range R in terms of f_r , f_m and Δf is given by :

$$\begin{aligned} R &= c f_r / 4f_m \Delta f = (3 \times 10^8) 20 \times 10^3 / 4(1 \times 10^6 \times 1 \times 10^3) \text{ mtrs} \\ &= 1500 \text{ mtrs} \\ &= 1.5 \text{ Kms} \end{aligned}$$

2. If the Radar operates at a frequency of 5GHZ, then find the Doppler frequency of an aircraft moving with a speed of 100KMph.

Solution:

Given,

The frequency of transmitted signal, $f=5\text{GHZ}$

Speed of aircraft (target), $V_r=100\text{KMph}$

$$\Rightarrow V_r = (100 \times 10^3) / 3600 \text{ m/sec} \Rightarrow V_r = 27.78 \text{ m/sec}$$

We know that, the speed of the light, $C=3 \times 10^8 \text{ m/sec}$

Now, following is the formula for Doppler frequency –

$$f_d = (2V_r f) / C$$

Substitute the values of V_r , f and C in the above equation.

$$\Rightarrow f_d = [2(27.78)(5 \times 10^9)] / 3 \times 10^8$$

$$\Rightarrow f_d = 926 \text{ HZ}$$

Therefore, the value of Doppler frequency, f_d is 926HZ for the given specifications.

Important Questions

1. What is the Doppler effect?
2. Explain the applications of CW Radar.
3. What are the factors that limit the amount of isolation between Transmitter and Receiver of CW Radar?
4. Explain the operation of the two frequency CW Radar.

5. With the help of a suitable block diagram explain the operation of a CW Radar with non zero IF amplifier.
6. Draw the block diagram of a FMCW Radar using side band super heterodyne receiver and explain it's operation.