Unit-1 : BASICS OF RADAR & RADAR EQUATION

Basic principles and features:

Radar is a contraction of the words Radio Detection And Ranging. Radar is an electromagnetic system for the detection and location of objects. It operates by transmitting a particular type of waveform, a pulse-modulated sine wave for example, and detects the nature of the echo signal.

- Radar can see through conditions such as darkness, haze, fog, rain, and snow which is not possible for human vision. In addition, radar has the advantage that it can measure the distance or range to the object.
- An elementary form of radar consists of a transmitting antenna emitting electromagnetic Radiation generated by an oscillator of some sort, a receiving antenna, and a signal receiver. A portion of the transmitted signal is intercepted by a reflecting object (target) and is reradiated in all directions. The receiving antenna collects the returned signal and delivers it to a receiver, where it is processed to detect the presence of the target and to extract its location and relative velocity. The distance to the target is determined by measuring the time taken for the Radar signal to travel to the target and back. The direction, or angular position, of the target is determined from the direction of arrival of the reflected wave front. The usual method of measuring the direction of arrival is with narrow antenna beams.
- If relative motion exists between target and radar, the shift in the carrier frequency of the reflected wave (Doppler Effect) is a measure of the target's relative (radial) velocity and may be used to distinguish moving targets from stationary objects. In radars which continuously track the movement of a target, a continuous indication of the rate of change of target position is also available. It was first developed as a detection device to warn the approach of hostile aircraft and for directing antiaircraft weapons. A well designed modern radar can extract more information from the target signal than merely range.

Measurement of Range:

- The most common radar waveform is a train of narrow, rectangular-shape pulses modulating a sine wave carrier.
- The distance, or range, to the target is determined by measuring the time TR taken by the pulse to travel to the target and return.
- Since electromagnetic energy propagates at the speed of light c (3 x 108 m/s) the range R is given by : R= cTR /2
- The factor 2 appears in the denominator because of the two-way propagation of radar. With the range R in kilometers or nautical miles, and TR in microseconds, the above relation becomes: R(km) = 0.15 X TR (µS)

(1 mile = 0.8689 nautical mile or
1.6 km 1 nautical mile = 1.15078 miles or
1.8412 km)

Maximum unambiguous range:

Once the transmitter pulse is emitted by the radar, sufficient time must elapse to allow any echo signals to return and be detected before the next pulse is transmitted. Therefore, the rate at which the pulses may be transmitted is determined by the longest range at which targets are expected. If the pulse repetition frequency is too high, echo signals from some targets might arrive after the transmission of the next pulse, and ambiguities in measuring range might result. Echoes that arrive after the transmission of the next pulse are called second-time-around (or multiple-time-around) echoes. Such an echo would appear to be at a much shorter range than the actual. The range beyond which targets appear as second-time-around echoes (or the farthest target range that can be detected by a Radar without ambiguity) is called the maximum unambiguous range and is given by: R unambig. = C/2fp Where fp = pulse repetition frequency, in Hz. (PRF)

This can also be explained with the following simple relations.

- TR is the time elapsed between transmission pulse and Echo pulse.
- TR = 2R/C where R= Range of target.
- TR increases with Range R and in extreme case Echo pulse merges with next Transmitted Pulse. Then TR becomes equal to TP Where TP= Pulse repetition period.
- TR max = TP= 2 R max /C and so R max = CTP/2 = C/2fP = R unambig.
- Therefore R unambig is directly proportional to the Pulse period TP (or Inversely proportional to the PRF fp).

Simple form of Radar Equation:

The radar equation

- Relates the range of a Radar to the characteristics of the transmitter, receiver, antenna, target, and environment.
- Useful as a means for determining the maximum measurable distance from the radar to the target
- It serves both as a tool for understanding radar operation and as a basis for radar design.

Derivation of the simple form of radar equation:

- If the power of the radar transmitter is denoted by Pt and if an isotropic antenna is used (one which radiates uniformly in all directions) the power density (watts per unit area) at a distance R from the radar is equal to the transmitter power divided by the surface area 4πR2of an imaginary sphere of radius R with radar at its centre, or

$$\text{Power density from anisotropic antenna} = P_t / 4\pi R^2$$

- Radars employ directive antennas to direct the radiated power Pt into some particular direction. The gain G of an antenna is a measure of the increased power radiated in the direction of the target as compared with the power that would have been radiated from an isotropic antenna. It may be defined as the ratio of the maximum radiation intensity from the given antenna to the radiation intensity from a lossless, isotropic antenna with the same power input. (The radiation intensity is the power radiated per unit solid angle in a given direction.)Then the power density at the target from an antenna with a transmitting gain G is given by

$$\text{Power density from directive antenna} = Pt.G/4\pi R^2$$

- The target intercepts a portion of the incident power and reradiates it in various directions. The measure of the amount of incident power intercepted by the target and reradiated back in the direction of the radar is denoted as the radar cross section σ, and is defined by the relation

$$\text{Power density of echo signal at radar} = (Pt\ .G/4\pi R^2\ )(\sigma)\ /4\pi R^2\ )$$

- The radar cross section σ has units of area. It is a characteristic of the particular target and is a measure of its size as seen by the radar. The radar antenna captures a portion of the echo power. If the effective area of the receiving antenna is denoted as Ae , then the power Pr .received by the radar is given by

$$Pr = (Pt\ .G/4\pi R^2\ ).(\ \sigma/4\pi R^2\ ).\ Ae = (Pt\ .G.\ Ae.\ \sigma)/\ (4\pi)^2\ .R^4$$

- The maximum radar range Rmax is the distance beyond which the target cannot be detected. It occurs when the received echo signal power Pr just equals the minimum detectable signal Smin. Therefore

$$Rmax = [\ (Pt\ .G.\ Ae.\ \sigma)/\ (4\pi)^2\ .\ Smin\ ]^{1/4}\ .....(1)$$

This is the fundamental form of the radar equation. Note that the important antenna parameters are the transmitting gain and the receiving effective area. Antenna theory gives the relationship between the transmitting gain and the receiving effective area of an antenna as:

$$G = 4\pi\ Ae\ /\lambda^2$$

Since radars generally use the same antenna for both transmission and reception, the above relation between gain G and affective aperture area Ae can be substituted into the above equation, first for Ae and then for G, to give two other forms of the radar equation.

$$Rmax = [\ (Pt\ .G^{2}\ .\ \lambda^{2}\ .\ \sigma)/\ (4\pi)^3\ .\ Smin\ ]^{1/4}\ ......(2)$$

$$Rmax = [\ (Pt\ .\ Ae^{2}\ .\ \sigma)/\ 4\pi.\ \lambda^2\ .\ Smin\ ]^{1/4}.......(3)$$

These three forms (eqs.1, 2, and 3) illustrate the need to be careful in the interpretation of the radar equation. For example, from Eq. (2) it might be thought that the range of radar varies as λ 1/2, but Eq. (3) indicates a λ -1/2relationship, and Eq. (1) shows the range to be independent of λ. The correct relationship depends on whether it is assumed the gain is constant or the effective area is constant with wavelength.

Limitations of the simple form of Radar equation:

- Does not adequately describe the performance of practical radar.
- Many important factors that affect range are not explicitly included.
- In practice, the observed maximum radar ranges are usually much smaller than what would be predicted by the above equations, sometimes by as much as a factor of two.

There are many reasons for the failure of the simple radar equation to correlate with actual performance and these will be explained subsequently in the modified Radar range equation.

Radar block diagram and operation:

The operation of a typical pulse radar is described with the help of a simple block diagram shown in the figure below.

- **Transmitter:** The transmitter is an oscillator, such as a magnetron, that is “pulsed” (turned on and off) by the modulator to generate a repetitive train of pulses. The magnetron has been the most widely used of the various microwave generators for radar. A typical radar for the detection of aircraft at ranges of 100 or 200 nmi employs a peak power of the order of one megawatt, an average power of several kilowatts, a pulse width of several microseconds, and a pulse repetition frequency of several hundred pulses per second.
- **Antenna:** The waveform generated by the transmitter travels via a transmission line to the antenna, where it is radiated into space. A single antenna is generally used for both transmitting and receiving.
- **Duplexer:** The receiver must be protected from damage caused by the high power of the transmitter. This is the function of the duplexer. The duplexer also serves to channel the returned echo signals to the receiver and not to the transmitter. The duplexer consists of two gas-discharge devices, one known as a TR (transmit-receive) and the other as ATR (anti-transmit-receive). The TR protects the receiver during transmission and the ATR directs the echo signal to the receiver during reception. Solid-state ferrite circulators and receiver protectors with gas-plasma TR devices and/or diode limiters are also employed as duplexers.
- **Receiver:** The receiver is usually of the super heterodyne type. The first stage normally is a low-noise RF amplifier, such as a parametric amplifier or a low- noise transistor.
- **Mixer:** The mixer and local oscillator (LO) convert the RF signal to an intermediate frequency IF. Typical IF amplifier center frequencies are 30 or 60 MHz and will have a bandwidth of the order of one megahertz.
- **IF amplifier :**The IF amplifier should be designed as a matched filter i.e., its frequency-response function H(f) should maximize the peak-signal-to-mean-noise-power ratio at the output. This occurs when the magnitude of the frequency-response function |H(f| is equal to the magnitude of the echo signal spectrum |S(f)|, and the phase spectrum of the matched filter is the negative of the phase spectrum of the echo signal. In a radar whose signal waveform approximates a rectangular pulse, the conventional IF filter band pass characteristic approximates a matched filter when the product of the IF bandwidth B and the pulse width τ is of the order of unity, that is, $B\tau = 1$.

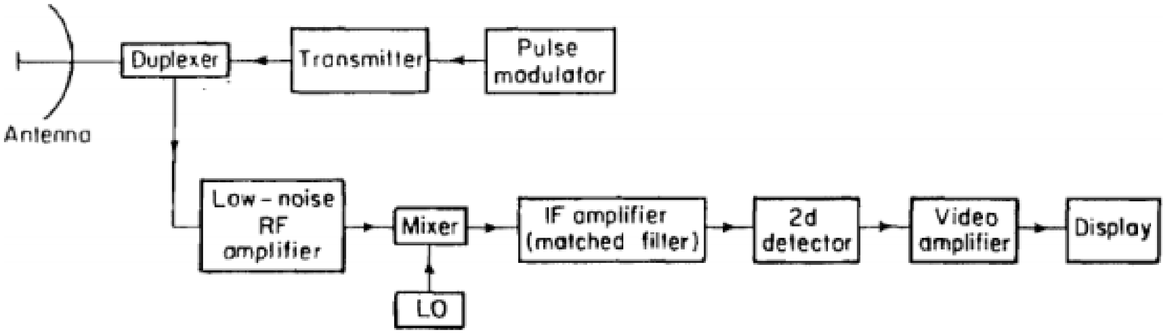
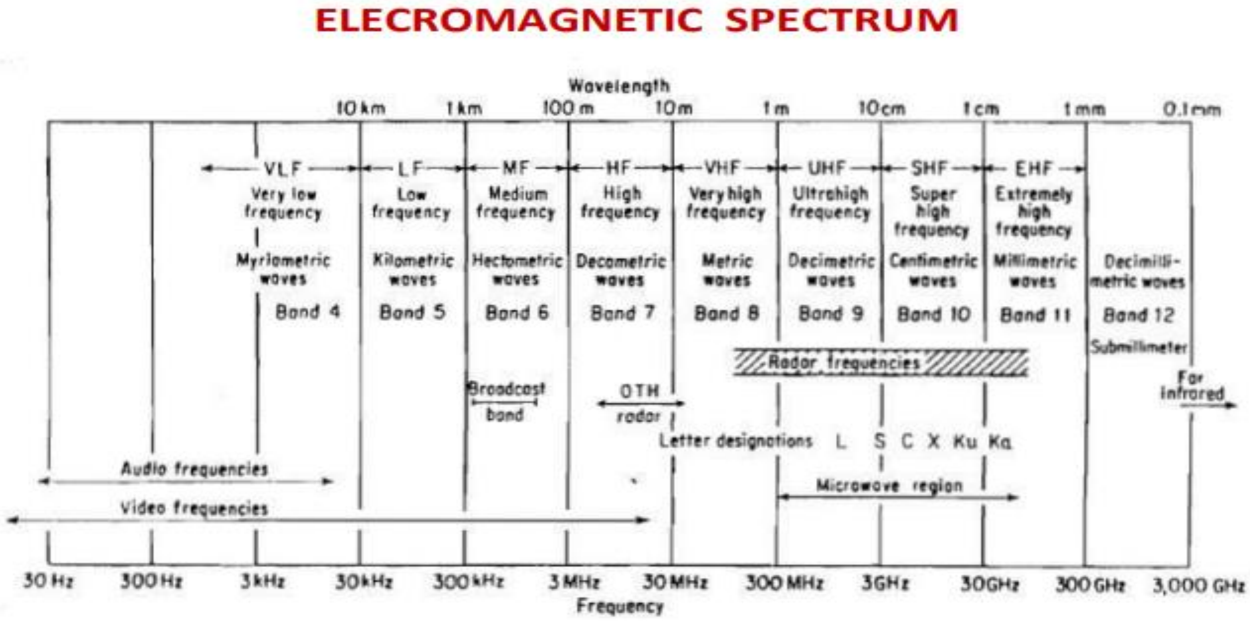


Fig. Block diagram of a pulse radar.

- **Second detector** : After maximizing the signal-to-noise ratio in the IF amplifier, the pulse modulation is extracted by the second detector and amplified by the video amplifier to a level where it can be properly displayed, usually on a cathode-ray tube (CRT). Timing signals are also supplied to the indicator to provide the range zero. Angle information is obtained from the pointing direction of the antenna.

Radar frequencies and applications:

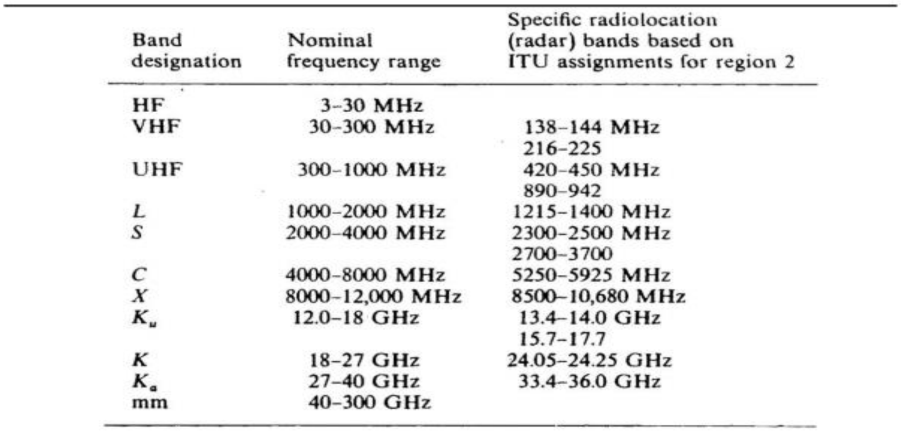
Radar frequencies: Conventional radars are operated at frequencies extending from about 220 MHz to 35 GHz, a spread of more than seven octaves. These are not necessarily the limits, since radars can be, and have been, operated at frequencies outside either end of this range. The place of radar frequencies in the electromagnetic spectrum is shown in the figure below. Some of the nomenclature employed to designate the various frequency regions is also shown in this figure.



Letter code designation of Radar frequencies:

Early in the development of radar, a letter code such as S, X, L, etc., was employed to designate Radar frequency bands. Although it's original purpose was to guard military secrecy, the designations were maintained, probably out of habit as well as the need for some convenient short nomenclature. This usage has continued and is now an accepted practice of radar engineers. The table below lists the radar-frequency letter-band nomenclature adopted by the IEEE. These are related to the specific bands assigned by the International Telecommunications Union for radar. For example, although the nominal frequency range for L band is 1000 to 2000 MHz, a L-band radar is thought of as being confined within the region from 1215 to1400MHz since that is the extent of the assigned band.

Table: Standard radar-frequency letter-band nomenclature



Band designation	Nominal frequency range	Specific radiolocation (radar) bands based on ITU assignments for region 2
HF	3-30 MHz	
VHF	30-300 MHz	138-144 MHz 216-225
UHF	300-1000 MHz	420-450 MHz 890-942
L	1000-2000 MHz	1215-1400 MHz
S	2000-4000 MHz	2300-2500 MHz 2700-3700
C	4000-8000 MHz	5250-5925 MHz
X	8000-12,000 MHz	8500-10,680 MHz
K_u	12.0-18 GHz	13.4-14.0 GHz 15.7-17.7
K	18-27 GHz	24.05-24.25 GHz
K_a	27-40 GHz	33.4-36.0 GHz
mm	40-300 GHz	

- **Applications of Radar:**

1. Military Use: Initial and important user of Radar

(i)Early warning of intruding enemy aircraft & missiles

(ii)Tracking hostile targets and providing location information to Air Defense systems consisting of Tracking Radars controlling guns and missiles.

(iii)Battle field surveillance

(iv)Information Friend or Foe IFF

(v)Navigation of ships, aircraft, helicopter etc.

2. Civilian Use:

(i)Air Traffic Control (ATC)

All airports are equipped with ATC Radars, for safe landing and take-off and guiding of aircraft in bad weather and poor visibility conditions.

(ii)Aircraft Navigation

(a) All aircrafts fitted with weather avoidance radars. These Radars give warning information to pilot about storms, snow precipitation etc. lying ahead of aircraft's path.

(b) Radar is used as an altimeter to indicate the height of the aircraft or helicopter.

3. Maritime ship's safety and Navigation:

(i)Radar used to avoid collision of ships during poor visibility conditions (storms, cyclones etc.) (ii)Guide ships into seaports safely.

4. Meteorological Radar:

Used for weather warnings and forecasting. Provides sufficient advance information to civilian administration for evacuation of population in times cyclones, storms etc.

Prediction of Range Performance:

The simple form of Radar equation derived earlier expresses the maximum radar range Rmax in terms of radar and target parameters:

$$R_{max} = [\ (P_t .G. A_e. \sigma)/\ (4\pi)^2 .S_{min}]^{1/4}$$

Where Pt = transmitted power, watts
G = antenna gain
Ae = antenna effective aperture, m2
σ = radar cross section, m2
Smin = minimum detectable signal, watts

All the parameters are to some extent under the control of the radar designer, except for the target cross section σ. The radar equation states that if long ranges are desired,

- The transmitted power must be large,
- The radiated energy must be concentrated into a narrow beam (high transmitting antenna gain),
- The received echo energy must be collected with a large antenna aperture (also synonymous with high gain) and
- The receiver must be sensitive to weak signals.

In practice, however, the simple radar equation does not predict the range performance of actual radar equipment to a satisfactory degree of accuracy. The predicted values of radar range are usually optimistic. In some cases, the actual range might be only half of that is predicted.

Part of this discrepancy is due to

- The failure of the above equation to explicitly include the various losses that can occur throughout the system or
- The loss in performance usually experienced when electronic equipment is operated in the field rather than under laboratory-type conditions and
- Another important factor i.e the statistical or unpredictable nature of several of the parameters in the radar equation.

  The minimum detectable signal Smin and the target cross section σ are both statistical in nature and must be expressed in statistical terms.
- Other statistical factors which do not appear explicitly in the simple radar equation but which have an effect on the radar performance are the meteorological conditions along the propagation path and the performance of the radar operator, if one is employed.

The statistical nature of these several parameters does not allow the maximum radar range to be described by a single number. Its specification must include a statement of the probability that the radar will detect a certain type of target at a particular range.

- Hence in order to cover these aspects, the simple radar equation will be modified to include most of the important factors that influence radar range performance.

Minimum detectable signal:

- The ability of a radar receiver to detect a weak echo signal is limited by the noise energy that occupies the same portion of the frequency spectrum as does the signal energy and accompanies the signal.
- The weakest signal the receiver can detect is called the minimum detectable signal. It is difficult to define minimum detectable signal (MDS) because of its statistical nature and because the criterion for deciding whether a target is present or not is not too well defined.
- Detection is normally based on establishing a threshold level at the output of the receiver (as shown by the dotted line in the figure below.)Whenever Rx output signal which is a mixture of echo and noise crosses this threshold, then it is detected as a target. This is called threshold detection.
- Consider the output of a typical radar receiver as a function of time as shown in the figure below which typically represents one sweep of the video output displayed on an A-scope.

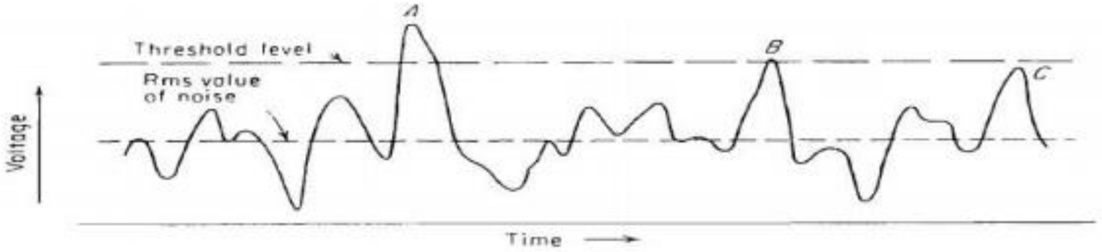


Fig. Typical envelope of the radar receiver output as a function of time. A, B, and C are three targets representing signal plus noise. A and B are valid detections, but C is a missed

- The envelope has a fluctuating appearance due to the random nature of noise and consists of three targets A, B and C of different signal amplitudes.
- The signal at A is large which has much larger amplitude than the noise. Hence target detection is possible without any difficulty and ambiguity.

- Next consider the two signals at B and C, representing target echoes of equal amplitude. The noise voltage accompanying the signal at B is large enough so that the combination of signal plus noise exceeds the threshold and target detection is still possible.
- But for the target C, the noise is not as large and the resultant signal plus noise does not cross the threshold and hence target is not detected.
- **Threshold Level setting:** Weak signals such as C would not be lost if the threshold level were lower. But too low a threshold increases the likelihood that noise alone will rise above the threshold and is taken as target. Such an occurrence is called a false alarm. Therefore, if the threshold is set too low, false target indications are obtained, but if it is set too high, targets might be missed. The selection of the proper threshold level is a compromise that depends upon how important it is if a mistake is made either by
    1. Failing to recognize a signal that is present (probability of a miss) or by
    2. Falsely indicating the presence of a signal when it does not exist (probability of a false alarm)
- The signal-to noise ratio necessary to provide adequate detection is one of the important parameters that must be determined in order to compute the minimum detectable signal.
- Although the detection decision is usually based on measurements at the video output, it is easier to consider maximizing the signal-to-noise ratio at the output of the IF amplifier rather than in the video. The receiver may be considered linear up to the output of the IF. It is shown that maximizing the signal-to-noise ratio at the output of the IF is equivalent to maximizing the video output. The advantage of considering the signal-to-noise ratio at the IF is that the assumption of linearity may be made. It is also assumed that the IF filter characteristic approximates the matched filter, so that the output signal-to-noise ratio is maximized.

Receiver noise:

Noise is unwanted electromagnetic energy which interferes with the ability of the receiver to detect the wanted signal thus limiting the receiver sensitivity.

It may originate within the receiver itself, or it may enter via the receiving antenna along with the desired signal. If the radar were to operate in a perfectly noise-free environment so that no external sources of noise accompanied the desired signal, and if the receiver itself were so perfect that it did not generate any excess noise, there would still exist an unavoidable component of noise generated by the thermal motion of the conduction electrons in the ohmic portions of the receiver input stages. This is called thermal noise, or Johnson's noise, and is directly proportional to the temperature of the ohmic portions of the circuit and the receiver band width. The available noise power generated by a receiver of bandwidth Bn(in hertz) at a temperature T (degrees Kelvin) is given by :

Available thermal-noise power = kTBn

where k = Boltzmann's constant =1.38 x 10-23J/deg.

If the temperature T is taken to be290 K, which corresponds approximately to room temperature (620F), the factor kT is 4 x 10--21W/Hz of bandwidth. If the receiver circuitry were at some other temperature, the thermal-noise power would be correspondingly different.

- A receiver with a reactance input such as a parametric amplifier need not have any significant ohmic loss. The limitation in this case is the thermal noise seen by the antenna and the ohmic losses in the transmission line.
- For radar receivers of the super heterodyne type (the type of receiver used for most radar applications), the receiver bandwidth is approximately that of the intermediate frequency stages. It should be cautioned that the bandwidth Bn mentioned above is not the 3-dB, or half-power, bandwidth commonly employed by electronic engineers. It is an integrated bandwidth and is given by:

  $$B_n = \frac{\int_{-\infty}^{\infty} |H(f)|^2 \, df}{|H(f_0)|^2}$$

  where H (f) = frequency-response characteristic of IF amplifier (filter) and

  fo = frequency of maximum response (usually occurs at mid band).

  - The bandwidth Bn is called the noise bandwidth and is the bandwidth of an equivalent rectangular filter whose noise-power output is same as the filter with characteristic H(f).It is not theoretically same as the 3-dB bandwidth. The 3-dB bandwidth is widely used since it is easy to measure. The measurement of noise bandwidth however involves a complete knowledge of the response characteristic H(f). The frequency response characteristics of many practical radar receivers are such that the 3 dB and the noise bandwidths do not differ appreciably. Therefore the 3-dB bandwidth may be used in many cases as an approximation to the noise bandwidth.
  - The noise power in practical receivers is often greater than can be accounted for by thermal noise alone and is due to mechanisms other than the thermal agitation of the conduction electrons. The exact origin of the extra noise components is not important except to know that it exists. Whether the noise is generated by a thermal mechanism or by some other mechanism the total noise at the output of the receiver may be considered to be equal to the thermal-noise power obtained from an "ideal" receiver multiplied by a factor called the noise figure.
  - The noise figure Fn of a receiver is defined by the equation:

Fn = No/ kTo BnGa

= (Noise output of practical receiver) / (Noise output of ideal receiver at std. temp To) Where No = noise output from receiver, and Ga = available gain. The standard temperature To is taken to be 290 K, according to the Institute of Electrical and Electronics Engineers definition. The noise No is measured over the linear portion of the receiver input-output characteristic, usually at the output of the IF amplifier before the nonlinear second detector. The receiver bandwidth Bn is that of the IF amplifier in most receivers. The available gain Ga is the ratio of the signal out So to the signal in Si and kTo Bn is the input noise Ni in an ideal receiver. The above equation may be rewritten as:

$$F_n = \frac{S_i/N_i}{S_o/N_o}$$

Therefore, the noise figure may be interpreted, as a measure of the degradation of signal-to noise-ratio as the signal passes through the receiver.

Modified radar equation:

Rearranging the above two equations for Fn , the input signal may be expressed as

$$S_i = \frac{kT_0 B_n F_n S_o}{N_o}$$

If the minimum detectable signal Smin is that value of Si corresponding to the minimum ratio of output (IF) signal-to-noise ratio (So/No)min necessary for detection, then

$$S_{\min} = kT_0 B_n F_n \left(\frac{S_o}{N_o}\right)_{\min}$$

Substituting this expression for Smin into the earlier basic Radar equation results in the following form of the modified radar equation:

$$R^4_{\max} = \frac{P_t G A_e \sigma}{(4\pi)^2 k T_0 B_n F_n (S_o/N_o)_{\min}} \qquad \text{.........(4)}$$

Signal to Noise Ratio (SNR):

The results of statistical noise theory will be applied to obtain:

- The signal-to-noise ratio at the output of the IF amplifier necessary to achieve a specified probability of detection without exceeding a specified probability of false alarm. The output signal-to-noise ratio thus obtained is substituted into the final modified radar equation, we have obtained earlier.

  The details of system that is considered:
- IF amplifier with bandwidth BIF followed by a second detector and a video amplifier with bandwidth BV as shown in the figure below.
- The second detector and video amplifier are assumed to form an envelope detector, that is, one which rejects the carrier frequency but passes the modulation envelope.
- To extract the modulation envelope, the video bandwidth must be wide enough to pass the low-frequency components generated by the second detector, but not so wide as to pass the high-frequency components at or near the intermediate frequency .
- The video bandwidth BV must be greater than BIF/2 in order to pass all the video modulation.

Integration of Radar Pulses:

The relation between the signal to noise ratio, the probability of detection and the probability of false alarm as shown in the figure or as obtained using the Albersheim's empirical equation applies for a single pulse only. However, many pulses are usually returned from any target on each radar scan and can be used to improve detection. The number of pulses nB returned from a point target as the radar antenna scans through its beam width is

$$\mathbf{n_B = \theta_B \;.\; f_P / \theta'_S = \theta_B \;.\; f_P / 6\,\omega_m}$$

where θB= antenna beam width, deg
fP= pulse repetition frequency, Hz
θ’S = antenna scanning rate, deg/s
ωm= antenna scan rate, rpm

The process of summing all the radar echo pulses for the purpose of improving detection is called integration.

Integration may be accomplished in the radar receiver either before the second detector (in the IF) or after the second detector (in the video).

- Integration before the detector is called pre detection or coherent integration. In this the phase of the echo signal is to be preserved if full benefit is to be obtained from the summing process.
- Integration after the detector is called post detection or non coherent integration. In this phase information is destroyed by the second detector. Hence post detection integration is not concerned with preserving RF phase. Due to this simplicity it is easier to implement in most applications, but is not as efficient as pre detection integration.

If n pulses, all of the same signal-to-noise ratio, were integrated by an ideal pre detection integrator, the resultant or integrated signal-to-noise (power) ratio would be exactly n times that of a single pulse. If the same n pulses were integrated by an ideal post detection device, the resultant signal-to-noise ratio would be less than n times that of a single pulse. This loss in integration efficiency is caused by the nonlinear action of the second detector, which converts some of the signal energy to noise energy in the rectification process.

Due to its simplicity, Post detection integration is preferred many a times even though the integrated signal-to-noise ratio may not be as high as that of Pre-detection. An alert, trained operator viewing a properly designed cathode-ray tube display is a close approximation to the theoretical post detection integrator.

The efficiency of post detection integration relative to ideal pre-detection integration has been computed by Marcum when all pulses are of equal amplitude. The integration efficiency may be defined as follows:

$$E_i(n) = \frac{(S/N)_1}{n(S/N)_n}$$

Where n = number of pulses integrated (S/N)1= value of signal-to-noise ratio of a single pulse required to produce a given probability of detection(for n = 1) (S/N)n = value of signal-to-noise ratio per pulse required to produce the same probability of detection when n pulses (of equal amplitude) are integrated .

The improvement in the signal-to-noise ratio when n pulses are integrated post detection is n. Ei(n) and is the integration-improvement factor. It may also be thought of as the effective number of pulses integrated by the post detection integrator. The improvement with ideal pre detection integration would be equal to n. Integration loss in decibels is defined as Li(n) = 10 log [1/Ei(n)]. The integration-improvement factor (or the integration loss) is not a sensitive function of either the probability of detection or the probability of false alarm. The radar equation with n pulses integrated can be written

$$R_{\max}^4 = \frac{P_t G A_e \sigma}{(4\pi)^2 k T_0 B_n F_n (S/N)_n}$$

where the parameters are the same as in the earlier Radar equation except that (S/N)n, is the signal-to noise ratio of one of the n equal pulses that are integrated to produce the required probability of detection for a specified probability of false alarm. Substituting the equation for integration efficiency

$$E_i(n) = \frac{(S/N)_1}{n(S/N)_n}$$

into the above Radar equation gives the final modified Radar equation including integration efficiency

$$R_{\max}^4 = \frac{P_t G A_e \sigma n E_i(n)}{(4\pi)^2 k T_0 B_n F_n (S/N)_1}$$

Radar Cross Section of Targets:

The radar cross section of a target is the (fictional) area intercepting that amount of power which when scattered equally in all directions, produces an echo at the radar equal to that from the target. Or in other terms

$$\sigma = \frac{\text{power reflected toward source/unit solid angle}}{\text{incident power density}/4\pi}$$

$$= \lim_{R \to \infty} 4\pi R^2 \left| \frac{E_r}{E_i} \right|^2$$

Where R= distance between radar and target

Er= strength of reflected field at radar

Ei= strength of incident field at target

For most common types of radar targets such as aircraft, ships, and terrain, the radar cross section does not necessarily bear a simple relationship to the physical area, except that the larger the target size, the larger will be the cross section.

Scattering and diffraction: are variations of the same physical process. When an object scatters an electromagnetic wave, the scattered field is defined as the difference between the total field in the presence of the object and the field that would exist if the object were absent (but with the sources unchanged). On the other hand, the diffracted field is the total field in the presence of the object. With radar backscatter, the two fields are the same, and one may talk about scattering and diffraction interchangeably.

Radar cross section of a simple sphere: is shown in the figure below as a function of its circumference measured in wavelengths.($2\pi a/\lambda$ where a is the radius of the sphere and λ is the wavelength). The plot consists of three regions.

1. Rayleigh Region 2. Optical region 3. Mie or Resonance region

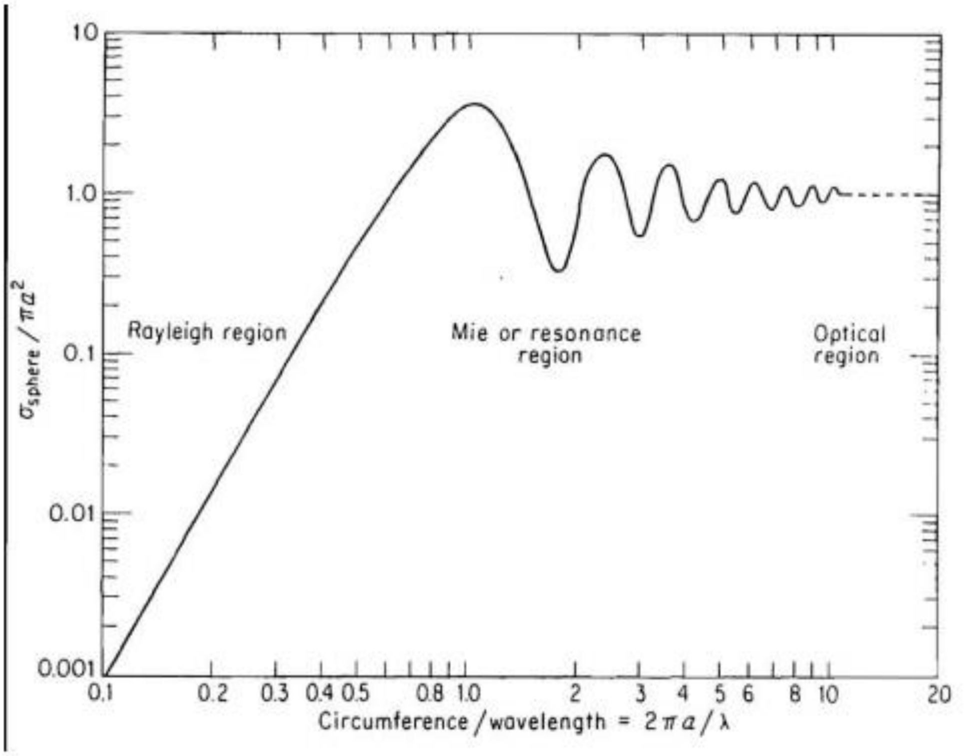


Fig.Radar cross section of the sphere. a = radius; λ= wavelength.

Radar cross section of a cone-sphere:

- An interesting radar scattering object is the cone-sphere, a cone whose base is capped with a sphere such that the first derivatives of the contours of the cone and sphere are equal at the joint. Figure below is a plot of the nose-on radar cross section. The cross section of the cone sphere from the vicinity of the nose-on direction is quite low.
- Scattering from any object occurs from discontinuities. The discontinuities, and hence the backscattering, of the cone-sphere are from the tip and from the join between the cone and the sphere.
- The nose-on radar cross section is small and decreases as the square of the wavelength. The cross section is small over a relatively large angular region. A large specular (having qualities

of a mirror)return is obtained when the cone-sphere is viewed at near perpendicular incidence to the cone surface, i.e., when θ= 90 - α, where α= cone half angle. From the rear half of the cone sphere, the radar cross section is approximately that of the sphere.

- The nose-on cross section of the cone-sphere varies, but its maximum value is approximately 0.4λ2 and its minimum is 0.01λ2 for a wide range of half-angles for frequencies above the Rayleigh region. The null spacing is also relatively insensitive to the cone half-angle.

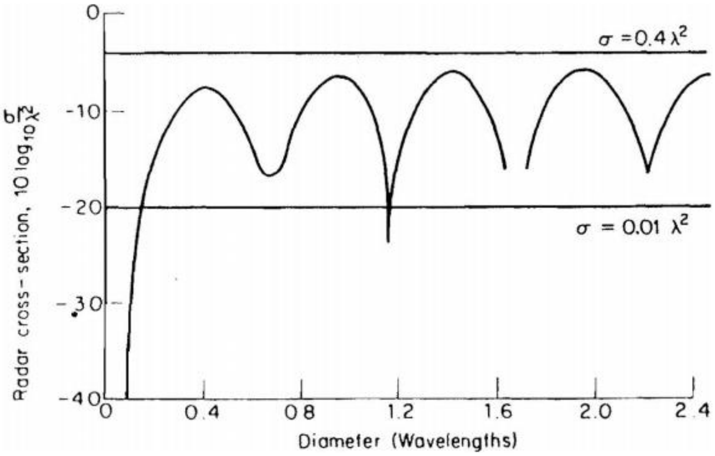


Fig. Radar cross section of a cone sphere with 150 half angle as a function of the diameter in Wave lengths.

- In order to realize in practice the very low theoretical values of the radar cross section for a cone sphere, the tip of the cone must be sharp and not rounded, the surface must be smooth (roughness small compared to a wavelength), the join between the cone and the sphere must have a continuous first derivative, and there must be no holes, windows, or protuberances on the surface.
- Shaping of the target, as with the cone-sphere, is a good method for reducing the radar cross section. Materials such as carbon-fiber composites, which are sometimes used in aerospace applications, can further reduce the radar cross section of targets as compared with that produced by highly reflecting metallic materials.

Transmitter Power:

The peak power: The power Pt in the radar equation is called the peak power. This is not the instantaneous peak power of a sine wave. It is the power averaged over that carrier-frequency cycle which occurs at the maximum power of the pulse.

The average radar power Pav: It is defined as the average transmitter power over the pulse-repetition period. If the transmitted waveform is a train of rectangular pulses of width τ and pulse-repetition period Tp = 1/ f p , then the average power is related to the peak power by

$$P_{av} = \frac{P_t \tau}{T_p} = P_t \tau f_p$$

Duty cycle: The ratio Pav/Pt, τ/TP, or τ.fP is called the duty cycle of the radar. A pulse radar for detection of aircraft might have typically a duty cycle of 0.001, while a CW radar which transmits continuously has a duty cycle of unity.

Writing the radar equation in terms of the average power rather than the peak power, we get

$$R^4_{max} = \frac{P_{av} G A_e \sigma n E_i(n)}{(4\pi)^2 k T_0 F_n (B_n \tau)(S/N)_1 \, f_p}$$

The bandwidth and the pulse width are grouped together since the product of the two is usually of the order of unity in most pulse-radar applications.

Pulse Repetition Frequencies and Range Ambiguities:

- The pulse repetition frequency (prf) is determined primarily by the maximum range at which targets are expected. If the prf is made too high, the likelihood of obtaining target echoes from the wrong pulse transmission is increased. Echo signals received after an interval exceeding the pulse-repetition period are called multiple time around echoes.
- Consider the three targets labeled A, B, and C in the figure(a) below. Target A is located within the maximum unambiguous range Runamb [= C.TP /2] of the radar, target B is at a distance greater than Runamb but less than 2Runamb and the target C is greater than 2Runamb but less than 3Runamb The appearance of the three targets on an A-scope is shown in the figure (b)below. The multiple-time-around echoes on the A-scope cannot be distinguished from proper target echoes actually within the maximum unambiguous range. Only the range measured for target A is correct; those for B and C are not.
- One method of distinguishing multiple-time-around echoes from unambiguous echoes is to operate with a varying pulse repetition frequency. The echo signal from an unambiguous range target will appear at the same place on the A-scope on each sweep no matter whether the prf is modulated or not. However, echoes from multiple-time-around targets will be spread over a finite range as shown in the figure (c) below. The number of separate pulse repetition frequencies will depend upon the degree of the multiple time around targets. Second-time targets need only two separate repetition frequencies in order to be resolved.

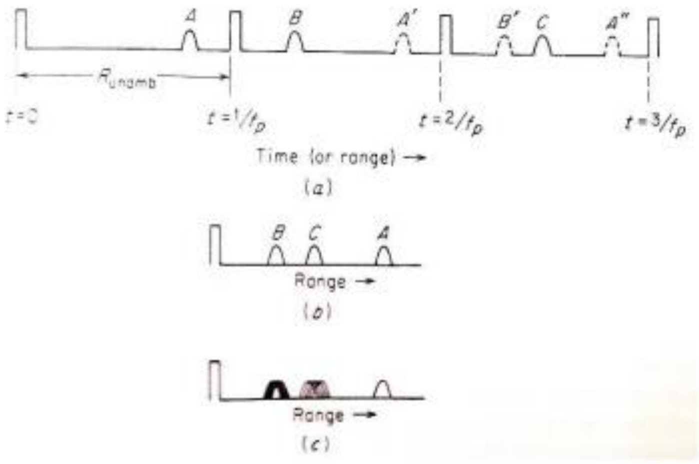


Fig. Multiple-time-around echoes that give rise to ambiguities in range. (a) Three targets A, B and C, where A is within Runamb, and B and Care multiple-time-around targets(b)the appearance of the three targets on the A-scope (c) appearance of the three targets on the A-scope with a changing prf.

System Losses:

- The losses in a radar system reduce the signal-to-noise ratio at the receiver output. They are two kinds, predictable with certain precision beforehand and unpredictable. The antenna beam shape loss, collapsing loss, and losses in the microwave plumbing are examples of losses which are predictable if the system configuration is known. These losses are real and cannot be ignored.
- Losses not readily subject to calculation and which are less predictable include those due to field degradation and to operator fatigue or lack of operator motivation. They are subject to considerable variation and uncertainty.

Plumbing loss: This is loss in the transmission lines which connects the transmitter output to the antenna. (Cables and waveguides). At the lower radar frequencies the transmission line introduces little loss, unless its length is exceptionally long. At higher radar frequencies, loss/attenuation will not be small and has to be taken into account.

Connector losses: In addition to the losses in the transmission line itself, additional losses occurs at each connection or bend in the line and at the antenna rotary joint if used. Connector losses are usually small, but if the connection is poorly made, it can contribute significant attenuation. If the same transmission line is used for both receiving and transmission, the loss to be inserted in the radar equation is twice the one-way loss.

Duplexer loss: The signal suffers attenuation as it passes through the duplexer. Generally, the greater the isolation required from the duplexer on transmission, the larger will be the insertion loss. Insertion loss means the loss introduced when the component is inserted into the transmission line. For a typical duplexer it might be of the order of 1 dB. In S-band (3000 MHz) radar, for example, the typical plumbing losses will be as follows:

100 ft of RG-113/U A1 waveguide transmission line (two-way): 1.0 dB
Loss due to poor connections (estimate): 0.5 dB Rotary-joint loss: 0.4 dB
Duplexer loss: 1.5 dB
Total plumbing loss: 3.4 dB

Beam-shape loss: The antenna gain that appears in the radar equation was assumed to be a constant equal to the maximum value. But in reality the train of pulses returned from a target with scanning radar is modulated in amplitude by the shape of the antenna beam. To properly take into account the pulsetrain modulation caused by the beam shape, the computations of the probability of detection (as explained earlier) would have to be performed assuming a modulated train of pulses rather than constant-amplitude pulses. But since this computation is difficult, a beam-shape loss is added to the radar equation and a maximum gain is employed in the radar equation rather than a gain that changes pulse to pulse.

Scanning loss: When the antenna scans rapidly enough , the gain on transmit is not the same as the gain on receive. An additional loss has to be computed, called the scanning loss. The technique for computing scanning loss is similar in principle to that for computing beam-shape loss. Scanning loss is important for rapid-scan antennas or for very long range radars such as those designed to view extraterrestrial objects.

Collapsing loss: If the radar were to integrate additional noise samples along with the wanted Signal-to noise pulses, the added noise results in degradation called the collapsing loss.

Non ideal equipment: The transmitter power in the radar equation was assumed to be the specified output power (either peak or average). However, all transmitting tubes are not uniform in quality, and even any individual tube performance will not be same throughout its useful life. Also, the power is not uniform over the operating band of frequencies. Thus, for one reason or another, the transmitted power may be other than the design value. To allow for this variation, a loss factor of about 2 dB is introduced.

Receiver noise figure also varies over the operating frequency band. Thus, if the best noise figure over the band is used in the radar equation, a loss factor has to be introduced to account for its poorer value elsewhere in the frequency band. If the receiver is not the exact matched filter for the transmitted waveform, a loss in Signal-to-noise ratio will occur. A typical value of loss for a non-matched receiver might be about 1 db. Because of the exponential relation between the false-alarm time and the threshold level a slight change in the threshold can cause a significant change in the false alarm time. In practice, therefore, it may be necessary to set the threshold level slightly higher than calculated so as to insure a tolerable false alarm time in the event of circuit instabilities. This increase in the threshold is equivalent to a loss.

Operator loss: An alert, motivated, and well-trained operator performs as described by theory. However, when distracted, tired, overloaded, or not properly trained, operator performance will decrease. The resulting loss in system performance is called operator loss.

Field degradation: When a radar system is operated under laboratory conditions by engineering personnel and experienced technicians, the above mentioned losses give a realistic description of the performance of the radar. However, when a radar is operated under field conditions the performance usually deteriorates even more than that can be accounted for by the above losses. To minimize field degradation Radars should be designed with built-in automatic performance-monitoring equipment. Careful observation of performance-monitoring instruments and timely preventative maintenance will minimize field degradation.

Illustrative Problems:

1) **A certain Radar has PRF of 1250 pulses per second. What is the maximum unambiguous range?**

Sol: Max. Unambiguous Range is given by

R unambig. = C /2fp

R unambig. = $3x10^8$ /2x1250 mtrs = $120X10^3$mts = 120Kms

2)A medium range search radar is working at 1.2GHz has a PRF of 300cps .Find unambiguous range in kms and in nautical miles and also find the wavelength of transmission.

[Note: C=3x10^8 m/sec and 1 nautical mile=1.852kms]

Solution: 1) Run (kms)= C/ (2fr)=500kms

2)Run (nmiles) =(500kmsX 1nmiles) /1. 852 nmi

=269. 978 nmiles

3) ƛ = C/ f = 0.25m

3) Calculate the maximum range of a radar system which operates at 3cm with a peak power 500kW. If the maximum receivable power is 10^-13W,the capture area of its antenna is 6m² and the radar cross section area of the target is 20m².

Solution: Given Data :- Pt= 500kW; λ = 3cm ($3X\ 10^{-2}$ m); Ae= $6m^2$; σ = $20m^2$; Pr=10^-13W

$$R_{max} = \left[(P_t . G . A_e . \sigma) / (4\pi)^2 . S_{min} \right]^{1/4}$$

Replace $G=(4\Pi A_e)/\lambda^2$

Ans: Rmax=750.94km

4)A tracing radar transmitter has a peak power of 400kW and PRF of 1500PPS and pulse width of 0.8μs. Calculate

a)Unambiguous range = C/2fr =(3X10^8)/2X1500 =100km
b) Duty Cycle = τ /T => τ X fr =(0.8X10^-6)(1500) =0.0012
c) Average power = peak power X duty cycle =400x10^3X0.0012 => 480W

Important questions:

1. Derive Radar range equation in terms of MDS (minimum detectable signal).
2. What is maximum unambiguous range? How is it related with PRF?
3. Explain the various system losses in a Radar.
4. Explain the basic principles of Radar and discuss about various parameters which improve the performance of the Radar.
5. Discuss about Radar frequencies and list out the Applications of Radars.
6. Explain how the Radar is used to measure the range of a target?
7. Draw the block diagram of the pulse radar and explain the function of each block.
8. Explain how a threshold level is selected in threshold detection?
9. Obtain the SNR at the output of IF amplifier of Radar Receiver for a specified probability of detection without exceeding a specified probability of false alarm.
10. Describe how pulse repetition frequency of a Radar system controls the range of it's detection?
11. Explain how the Transmitted power affects the range.
12.Distinguish between Monostatic and Bistatic Radars.
13. Explain the radar cross section of the sphere.